Map of the USA
Exploring the Addition and Subtraction of Vectors

About this Lesson
This activity provides practice for students in vector operations as they are used in navigation. It also provides a visual experience so that the operations with vectors become better understood and less abstract.

This lesson is included in LTF Physics Module 5.

Objective
Students will use a map of the US to:

- Study the addition of displacement vectors to locate positions.
- Use velocity vectors to calculate time.

Level
Physics

Common Core State Standards for Science Content

LTF Science lessons will be aligned with the next generation of multi-state science standards that are currently in development. These standards are said to be developed around the anchor document, *A Framework for K–12 Science Education*, which was produced by the National Research Council. Where applicable, the LTF Science lessons are also aligned to the Common Core Standards for Mathematical Content as well as the Common Core Literacy Standards for Science and Technical Subjects.

Code	Standard	Level of Thinking	Depth of Knowledge
(LITERACY) RST.9-10.3	Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.	Apply	II
(LITERACY) RST.9-10.7	Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.	Apply	II
(MATH) A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.	Apply	II
(MATH) F-LE.5	Interpret expressions for functions in terms of the situation they model. Interpret the parameters in a linear or exponential function in terms of a context.	Apply	II

Code	Standard	Level of Thinking	Depth of Knowledge
(MATH) A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R.	Apply	II
(MATH) N-Q.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	Apply	II

Connections to AP

I. Newtonian mechanics, A. Kinematics (including vectors and vector algebra, components of vectors, coordinate systems, displacement, velocity, and acceleration) 2. Motion in two

Materials

Each lab group will need the following:

calculator, TI® graphing
meter stick
protractor, with hole
ruler, clear metric
scissors
tape, clear plastic

Assessments

The following types of formative assessments are embedded in this lesson:

- Assessment of prior knowledge
- Guided questions
- Assessment of process skills

The following additional assessments are located on the LTF website:

- Physics Assessment: Newton's Laws of Motion
- 2009 Physics Posttest, Free Response Question 2

Teaching Suggestions

Any map of the US will work. However, it is best if everyone has the same map. Maps will be provided and may be printed for classroom use.

The map provided is printed as a flat map and therefore the measured distances on this map will not exactly represent the actual distances if measured along the curved surface of the earth. Also since the map is flattened, the directions indicated by the grid also do not represent true directions with respect to N, this can be noted by looking at the compass legend and the grid lines shown on the map. The distances and directions that are given in the exercise therefore do not represent actual distances or directions that must be flown by an airplane to reach each destination.

Students should use a line drawn north/south and east/west through Dallas, Texas as a direction reference for this exercise. Use the lines which outline the map as a north/south, east/west reference. But keep in mind, that these directions are not exactly correct, because the map is actually a section from the surface of a round Earth. Students should arrange the protractor so that the center of the protractor is over the given destination and the straight edge of the protractor is parallel to the "East-West" line drawn through Dallas.

PART I: DISPLACEMENT

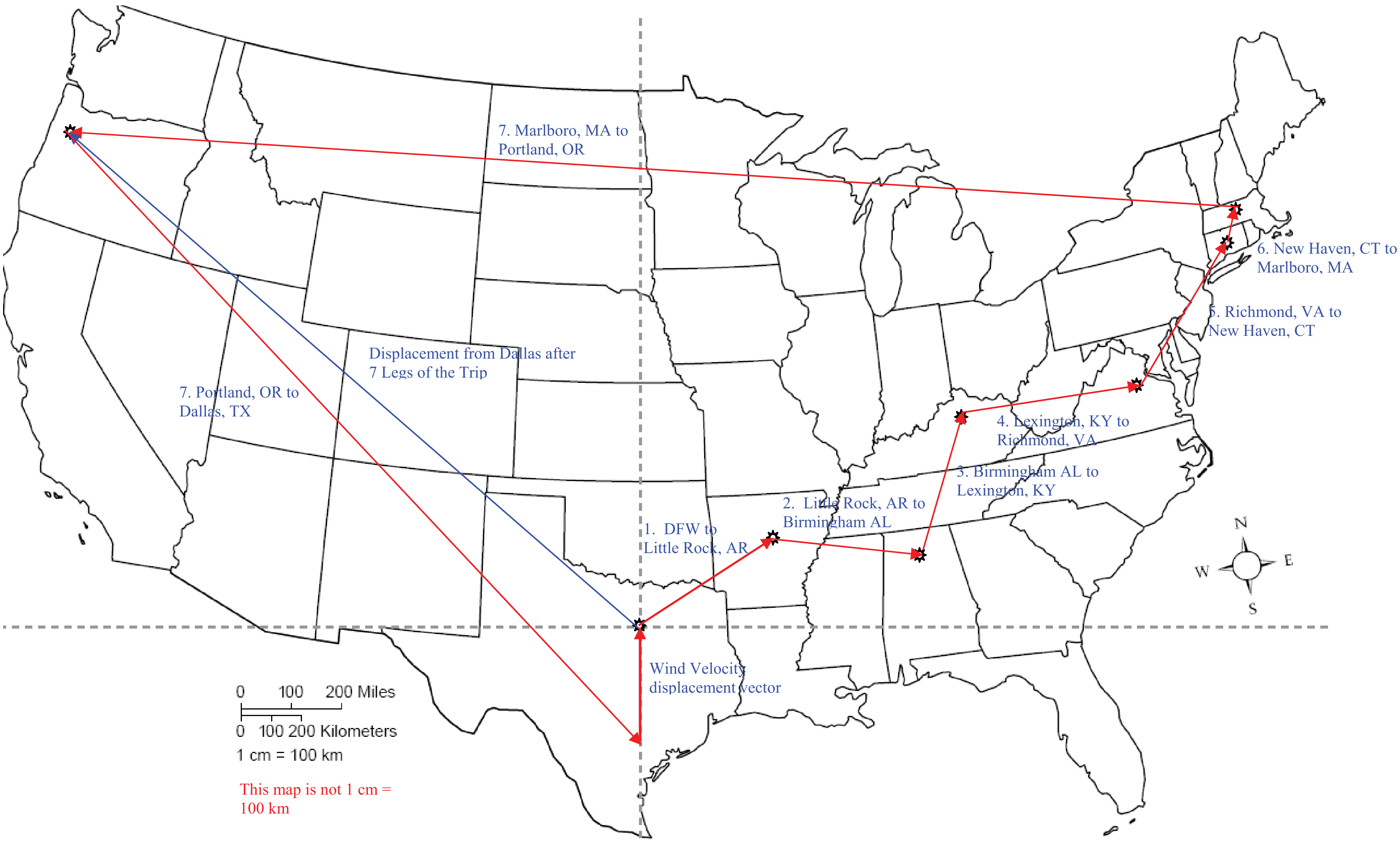
In each of the following steps, calculate the displacement of the airplane and draw the appropriate displacement vector on the map provided by your teacher. Be sure to add each vector "head-to-tail" to the vector that precedes it.

1. Starting from DFW (Dallas-Fort Worth) airport, an airplane flies at a heading of N 54° E or 306° at a speed of 420 km/hr for 1.25 hours.
   Closest airport is Little Rock, AR

2. After visiting the Clinton Library, the airplane travels 480 km at a heading of N 98° E. As the pilot approaches to land, he sees the Vulcan statue.
   Closest airport is Birmingham, AL

3. Next the airplane travels at a speed of 625 kilometers per hour for 45 minutes at a heading of N 22° E. After the pilot lands, he pays homage to the likes of Secretariat, Seattle Slew, Affirmed and 8 others before returning to his trip.
   Closest airport is Lexington, KY

4. Next, the plane travels 570 km at a heading of N 84° E. While taking a break, the pilot visits a state capitol building designed by Thomas Jefferson and enjoys the city which was once the home of Pocahontas. Closest airport is Richmond, VA
5. After enjoying a nice ham dinner, the plane flies at 410 km/hr for 1.3 hrs at a heading of N 35° E. The pilot enjoys a nice aerial view of Yale University and stops to refuel before continuing his trip. Closest airport is New Haven, CT
6. After resting and refueling the plane proceeds 130 km at N 21 ° E. The pilot decides to spend the night and enjoys a fantastic dinner of lobster and clams before beginning a long westward flight. Closest airport is Marlboro, MA
7. This last destination before returning home is 3770 km at N 85° W. After landing the pilot takes the MAX light rail to visit his friends at Vernier Scientific and Software in nearby Beaverton. After visiting the Vernier Solar Energy Demonstration Project, the pilot returns to his hotel to contemplate his trip and plan a return home to Dallas, Texas.
   Closest airport is Portland, OR


7. Marlboro, MA to Portland, OR
6. New Haven, CT to Marlboro, MA
5. Richmond, VA to New Haven, CT
Displacement from Dallas after 7 Legs of the Trip
7. Portland, OR to Dallas, TX
4. Lexington, KY to Richmond, VA
3. Birmingham AL to Lexington, KY
2. Little Rock, AR to Birmingham AL
1. DFW to Little Rock, AR
N
W
E
S
Wind Velocity displacement vector
0 100 200 Miles
0 100 200 Kilometers
1 cm = 100 km
This map is not 1 cm = 100 km

PART: II VELOCITY

Use the same map, but use a different colored marker to distinguish between the vectors in Part I and Part II. Use the same scale as in Part I. Draw the appropriate vectors indicated.

After returning to his hotel near the Willamette River the pilot takes out his map and contemplates his trip and his anticipated return home.

1. Construct the resultant of all the legs of the trip which resulted in the pilots visit to Portland, Oregon.

   - On the map, a vector drawn from DFW to Portland, Oregon

2. The equilibrant is a vector equal and magnitude and opposite in direction to the resultant. Reverse the direction of the resultant by placing an arrow on the head end of the resultant. This now represents the desired ground speed vector the plane must achieve to return to Dallas Love Field.

   - The same arrow constructed in #1 with the arrow head reversed to show the equilibrant which is equal and opposite of the resultant. e.g. drawn from Portland, Oregon to Dallas, Texas.

3. The pilot wishes to depart the next morning from the airport in Portland, Oregon, and fly to the airport nearest downtown Dallas (Love Field). The pilot wishes to make this trip in 4.5 hours. There is a constant wind of 55 kilometers per hour blowing N 0° (due north or from the south). How far will the wind travel during this 4.5 hour trip? Draw a wind vector starting this distance south of Dallas and ending at Love Field. This represents your wind speed vector.

   - 55 km/h × 4.5 h = ~ 250 km

   - An arrow drawn from the south of Dallas (approximately 2.5 cm) and terminating with the arrow head on Dallas, Tx. See map opposite page 34 in Chemistry/Physics manual or opposite page page 38 in the Physics manual

4. Now draw a vector from Portland to the tail of the wind speed vector. This represents the plane’s required air speed vector. The resultant of the plane’s air speed vector and the wind speed vector gives the plane’s ground speed vector.

   - See the map

5. Label these vectors on your map.

   - See the map

CONCLUSION QUESTIONS

PART I: DISPLACEMENT

1. What is the distance traveled by the airplane in the first leg of the trip?

    - 480 km

2. At the end of each leg the pilot lands at the nearest airport. What is the name of the town nearest to the airport at the end of the second leg?

    - Birmingham, AL

3. What is the total distance flown for the entire trip before returning from Portland?

    - 525 km + 480 km + 470 km + 570 km + 530 km + 130 km + 3770 km = 6475 km

4. After the 7 legs of the trip, what is the total displacement from DFW airport (both magnitude and direction)? Draw this vector on your map.

    - 2380 km at N 129° E

5. Between what two states does the Mississippi run where the plane crosses the river?

    - Arkansas and Mississippi

6. What is the name of the city or town nearest to the airport when the plane lands at its eastern most point?

    - Marlboro, MA

PART II: VELOCITY

1. What is the bearing (direction relative to north) for the vector between the airport at Portland, Oregon and the airport Dallas Love Field?

    - N 131 ° E

2. How far is the pilot from Dallas, Texas?

    - 2450 Km

3. The pilot wishes to depart the next morning from the airport in Portland, Oregon, and fly to the airport nearest downtown Dallas (Love Field). The pilot wishes to make this trip in 4.5 hours. There is a constant wind of 55 kilometers per hour blowing due north (from the south). What air speed in kilometers per hour and what direction relative to north must the pilot maintain in order to complete the trip in 4.5 hours?

    - The pilot must fly 2620 km at N 135 ° E. In order to accomplish this in 4.5 hours, the pilot must fly at approximately 580 km/h.

    - When added to the wind vector of 250 km at 0 °, due N the sum gives the resultant of 2450 km at
      N 131 ° E, and the pilot will be at Love Field, Dallas in approximately 4.5 hrs.

4. Indicate and label all three vectors (air, wind and ground) on your map. Indicate both the magnitude and direction of each vector.

    - 2620 km at N, 135 ° E in 4.5 hr gives the air speed vector, 582 km/h at 135 ° E
    - 250 km at N, 0 ° in 4.5 hr gives the wind speed vector, 55 km/h at N, 0 °
    - 2450 km at N 131 ° E in 4.5 hr gives the resultant or ground speed vector, 540 km/h at N 131 ° E

5. What is the scale which is represented by these velocity vectors?

    - The scale for these velocity vectors are approximately 1 cm = 22 km/h

      $$\frac{582km/h}{26.2cm} = \frac{22.2km/h}{cm}$$
      $$\frac{55km/h}{2.5cm} = \frac{22km/h}{cm}$$

Map of the USA
Exploring the Addition and Subtraction of Vectors

The information below describes a series of flights taken by a pilot. In Part I you will draw, as precisely as possible, the displacement vectors of this flight on your map. In Part II, you will use the same map to draw several velocity vectors on the map and determine the airplane's velocity. You should first draw and label your vectors in pencil. After you complete the preliminary work, trace all of your vectors and label the magnitudes and angles in marker so that they are clearly visible.

Remember: $\vec{v}_{ground} = \vec{v}_{air} + \vec{v}_{wind}$ or $\vec{v}_{air} = \vec{v}_{ground} - \vec{v}_{wind}$

Note:
The scale of this map is shown on the map. It was originally printed such that 1.00 cm = 100 Kilometers, however, in reproducing maps, they do not always stay to the original scale.

Longitude and latitude lines are not shown on the map, and directions and distances will not be absolutely correct because of the curvature of the earth.

PURPOSE
You will use a map of the USA to study the addition and subtraction of vectors. The more careful you are in measuring the angles and the displacements, the better your results will be.

MATERIALS

Each lab group will need the following:

calculator, TI® graphing
meter stick
protractor, with hole
ruler, clear metric
scissors
tape, clear plastic

PROCEDURE

PART I: DISPLACEMENT

In each of the following steps, calculate the displacement of the airplane and draw the appropriate displacement vector on the map provided by your teacher. Be sure to add each vector “head-to-tail” to the vector that precedes it.

1. Starting from DFW (Dallas-Fort Worth) airport, an airplane flies at a heading of N 54^o E or 306^o at a speed of 420 km/hr for 1.25 hours.

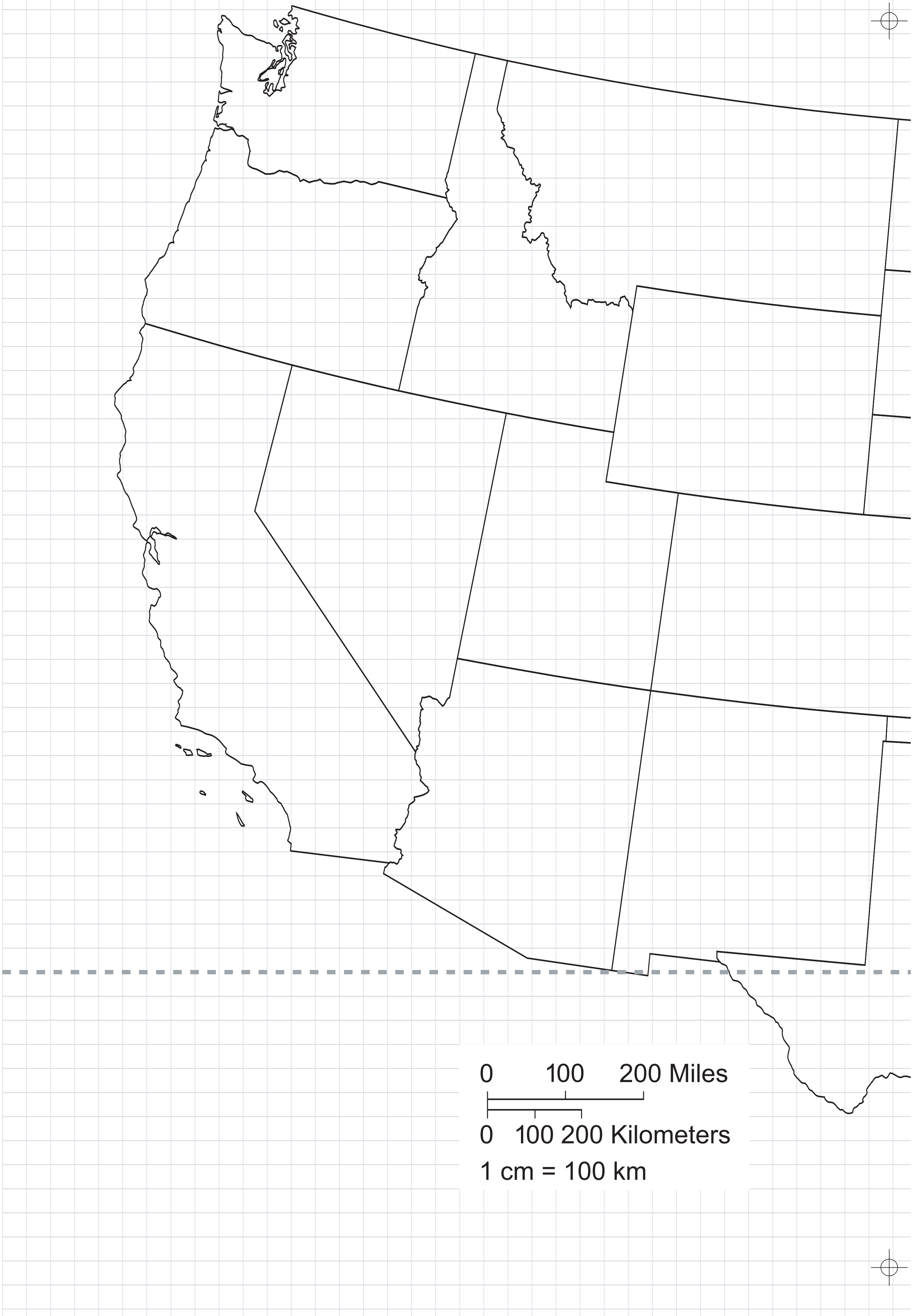
2. After visiting the Clinton Library, the airplane travels 480 km at a heading of N 98^o E. As the pilot approaches to land, he sees the Vulcan statue.

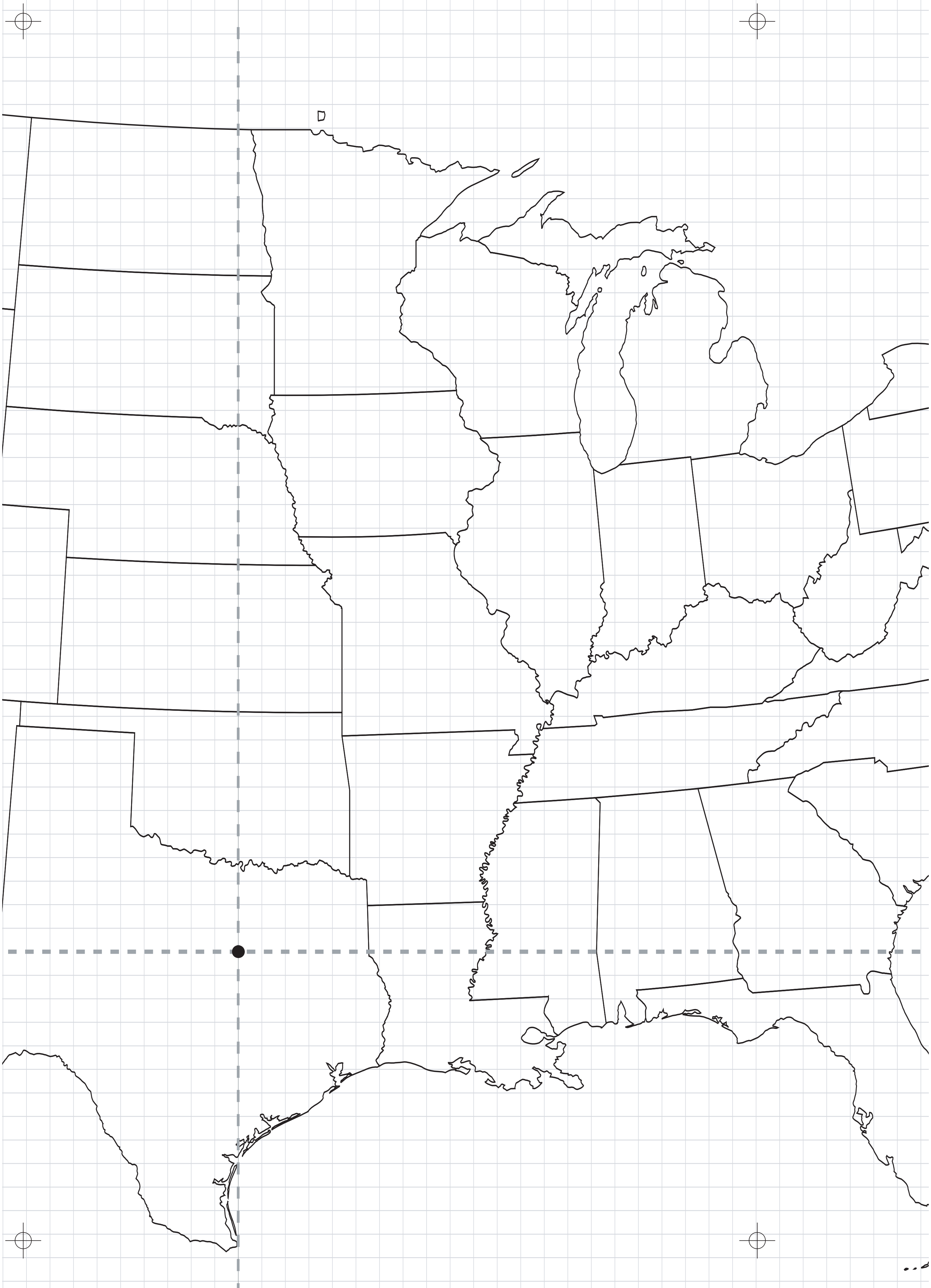
3. Next the airplane travels at a speed of 625 kilometers per hour for 45 minutes at a heading of N 14^o E. After the pilot lands, he pays homage to the likes of Secretariat, Seattle Slew, Affirmed and 8 others before returning to his trip.

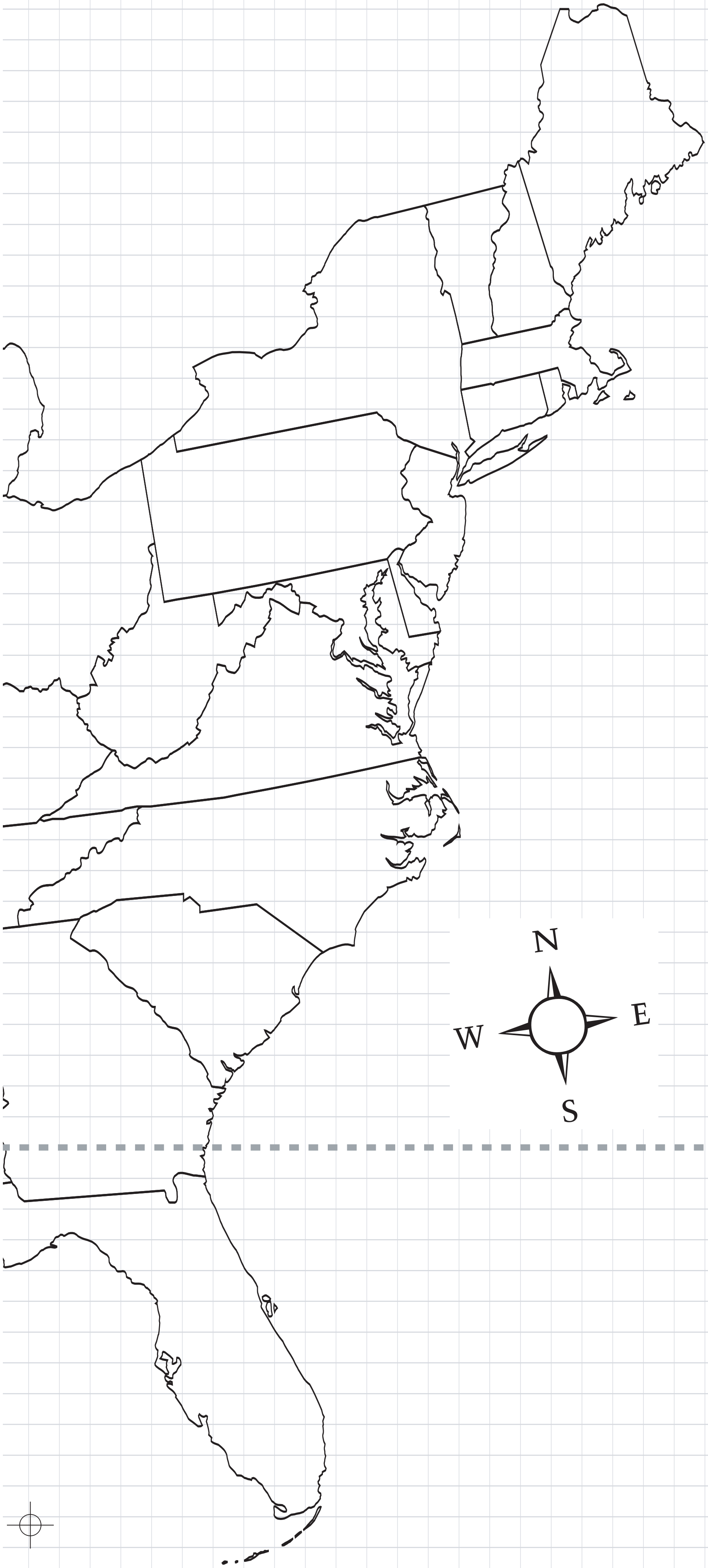
4. Next, the plane travels 570 km at a heading of N 84^o E. While taking a break, the pilot visits a state capitol building designed by Thomas Jefferson and enjoys the city which was once the home of Pocahontas.

5. After enjoying a nice ham dinner, the plane flies at 410 km/hr for 1.3 hrs at a heading of N 35° E. The pilot enjoys a nice aerial view of Yale University and stops to refuel before continuing his trip.

6. After resting and refueling the plane proceeds 130 km at N 21 ° E. The pilot decides to spend the night and enjoys a fantastic dinner of lobster and clams before beginning a long westward flight.
7. This last destination before returning home is 3770 km at N 85° W^o. After landing the pilot takes the MAX light rail to visit his friends at Vernier Scientific and Software in nearby Beaverton. After visiting the Vernier Solar Energy Demonstration Project, the pilot returns to his hotel to contemplate his trip and plan a return home to Dallas, Texas.


0 100 200 Miles
0 100 200 Kilometers
1 cm = 100 km




N
W
E
S

PART II: VELOCITY
Use the same map, but use a different colored marker to distinguish between the vectors in Part I and Part II. Use the same scale as in Part I. Draw the appropriate vectors indicated.

After returning to his hotel near the Willamette River the pilot takes out his map and contemplates his trip and his anticipated return home.

1. Construct the resultant of all the legs of the trip which resulted in the pilots visit to Portland, Oregon.

2. The equilibrant is a vector equal and magnitude and opposite in direction to the resultant. Reverse the direction of the resultant by placing an arrow on the head end of the resultant. This now represents the desired ground speed vector the plane must achieve to return to Dallas Love Field.

3. The pilot wishes to depart the next morning from the airport in Portland, Oregon, and fly to the airport nearest downtown Dallas (Love Field). The pilot wishes to make this trip in 4.5 hours. There is a constant wind of 55 Kilometers per hour blowing N 0° (due north or from the south). How far will the wind travel during this 4.5 hour trip? Draw a wind vector starting this distance south of Dallas and ending at Love Field. This represents your wind speed vector.

4. Now draw a vector from Portland to the tail of the wind speed vector. This represents the plane's required air speed vector. The resultant of the plane's air speed vector and the wind speed vector gives the plane's ground speed vector.

5. Label these vectors on your map.

Map of the USA
Exploring the Addition and Subtraction of Vectors

DATA AND OBSERVATIONS

All of your work and results should be shown on the map.

CONCLUSION QUESTIONS

PART I: DISPLACEMENT

1. What is the distance traveled by the airplane in the first leg of the trip?

2. At the end of each leg the pilot lands at the nearest airport. What is the name of the town nearest to the airport at the end of the second leg?

3. What is the total distance flown for the entire trip before returning from Portland?

4. After the 7 legs of the trip, what is the total displacement from DFW airport (both magnitude and direction) Draw this vector on your map.

5. Between what two states does the Mississippi run where the plane crosses the river?

6. What is the name of the city or town nearest to the airport when the plane lands at its eastern most point?

PART II: VELOCITY

1. What is the bearing (direction relative to north) for the vector between the airport at Portland, Oregon and the airport Dallas Love Field?

2. How far is the pilot from Dallas, Texas?

3. The pilot wishes to depart the next morning from the airport in Portland, Oregon, and fly to the airport nearest downtown Dallas (Love Field). The pilot wishes to make this trip in 4.5 hours. There is a constant wind of 55 kilometers per hour blowing due north (from the south). What air speed in kilometers per hour and what direction relative to north must the pilot maintain in order to complete the trip in 4.5 hours?

4. Indicate and label all three vectors (air, wind and ground) on your map. Indicate both the magnitude and direction of each vector.

5. What is the scale which is represented by these velocity vectors?