Transformations and Tessellations

ABOUT THIS LESSON

In this lesson, students explore translations and reflections from tilings and tessellations. They then determine the area of specific tiles and use a scale model to calculate the cost of tiling a floor. The lesson also introduces the process of tiling and includes a hands-on activity so that students can discover a rule to determine the properties of regular polygons that will tessellate. Finally, students explore a tessellation originally formed from a rectangle and realize that as the rectangle is modified to make, in this case, a ghost, the area of the rectangle is preserved.

OBJECTIVES

Students will

- create a tiling.
- develop a rule to determine which regular polygons will tessellate.
- design a tessellation using translations.
- create a tessellation using rotations.

LEVEL

Grade 8 in a unit on areas of polygons or transformations

MODULE/CONNECTION TO AP*

Analysis of Functions: Transformations

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MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards

8.G.1: Verify experimentally the properties of rotations, reflections, and transformations.
See questions 2, 8-9, 11, 14-15, 17

8.G.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. *For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.*
See question 10

Reinforced/Applied Standards

7.G.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
See questions 3-5, 16

7.G.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
See questions 3-5

7.EE.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. *For example: If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.*
See questions 3-6, 12

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.1: Make sense of problems and persevere in solving them.
Students label the given information for Pattern A, understand how the shapes are put together, and determine the lengths of the sides, the area of each shape, the area of the tiling, the number of whole tiles that are required, and the total cost of the floor with Pattern A and Pattern B.

MP.7: Look for and make use of structure.
Students analyze the shapes that compose each pattern, draw in auxiliary lines, and apply geometric properties.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Use given information to calculate dimensions in a composite figure
- Calculate the areas of triangles and trapezoids

ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on our website:

- Analysis of Functions: Transformations – 7th Grade Free Response Questions
- Analysis of Functions: Transformations – 7th Grade Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
- Straight edges
- Scissors
- Tape
- Patty paper or tracing paper
- Cardstock
- Colored pencils
- Pattern blocks (optional)
- Applets for creating a tiling using pattern blocks: http://nlvm.usu.edu/en/nav/frames_asid_171_g_3_t_3.html?open=activities&from=category_g_3_t_3.html

TEACHING SUGGESTIONS

This lesson provides an opportunity for students to explore transformations in the context of a tiling. The first part of the lesson can be used to identify and work with shapes as a review of basic geometry. In questions 3 – 6, students determine area of specific tiles and calculate the cost of a tiling based on a scale drawing. They use these computations in question 7 to determine whether or not the tiling is within the given budget. For question 8 – 9, students may use patty paper to trace the original figure and rotate the patty paper to draw the congruent polygons at the vertex to see if it forms a tessellation.

The second section of the lesson focuses on tessellations formed by regular polygons. Question 10 provides an opportunity for students to work with interior and exterior angles of polygons. To help students visualize the answers, show how to divide each figure into triangles to calculate the sum of the interior angles. This method of determining the sum of the interior angles is a forerunner to the formula, $180°(n - 2)$, which is used in geometry.

The last part of the lesson involves tessellations. Each student will make a unique shape that will tessellate using translations and/or rotations. In addition, students will explore what happens to that shape as a result of their transformations. As an extension, this topic makes an excellent interdisciplinary lesson with students' language arts or computer literacy classes. The tessellations can also be created using a computer. Student projects can include the definition and origin of the word tessellation, the history of tessellations, tessellations in nature, and tessellations in art history from ancient architecture to modern art. Any student project should include a study of M. C. Escher's tessellations.

Suggested modifications for additional scaffolding include the following:

2	Label the given information and the bases of the trapezoids on Pattern A.
3,4	Provide a table to organize the calculations of the area which may include "Number of Each Tile in the Pattern, Tile, Formula to Calculate the Area, Process Column for Calculation, Area of each tile, Sum of the Area for the Shape."
10	Complete the row of the table for a pentagon and/or an octagon.
14	Provide a tiling or a tessellation that has been started for the student to complete.

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Apply transformations to tessellations as well as to points, segments, and figures on the coordinate plane.	Apply transformations to tessellations as well as to points, segments, and figures on the coordinate plane.	Apply transformations including $a\,f(x-c)+d$ to linear, quadratic, exponential, piecewise, and generic functions.	Apply transformations to circles and apply transformations including $a\,f(x-c)+d$ to linear, quadratic, exponential, piecewise, and generic functions.	Apply transformations to conic sections and apply transformations including $a\,f(x-c)+d$ and compositions with absolute value including $f(\|x\|)$ and $\|f(x)\|$ to parent, piecewise, and generic functions.	Apply transformations to conic sections and apply transformations including $a\,f(x-c)+d$ and compositions with absolute value including $f(\|x\|)$ and $\|f(x)\|$ to linear, polynomial, exponential, logarithmic, trigonometric, piecewise, and generic functions.

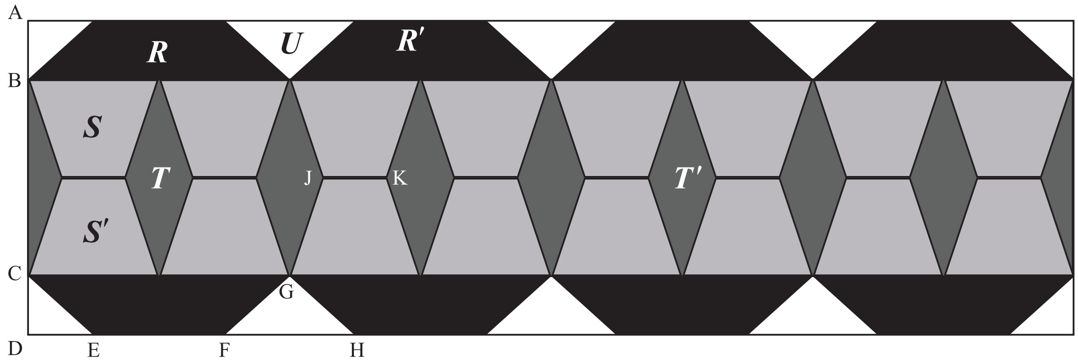
Transformations and Tessellations

Throughout history, artists and architects have covered flat surfaces with tiles. Tile designs can be as simple as the square tiles covering a shower wall or as complex as a stained glass window. The only limitation on tiling designs is that the tiles must be arranged so that no empty space is left between the tiles.

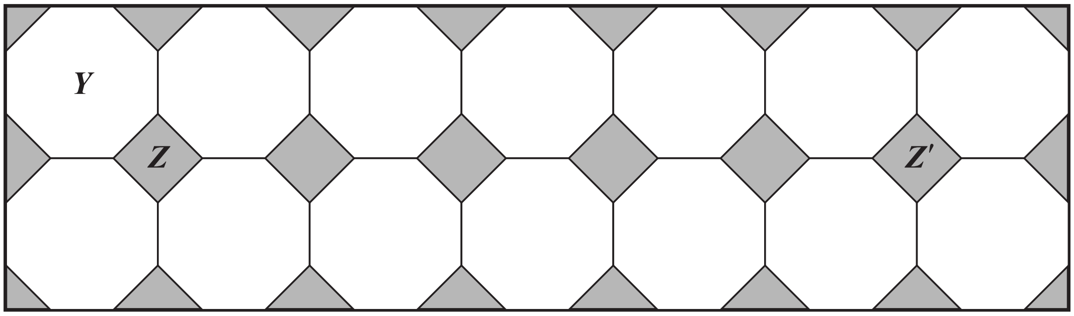
The McAlister's are planning to build a new house. Their architect has proposed two patterns for tiling the kitchen floor. Below are the scaled models, each covering the same area, of the tile patterns for them to consider. They have a budget of $850 to spend on the floor tiles.

Pattern A is created using shapes *R, S, T*, and *U*. To completely fill the rectangle shown, shapes *T* and *U* are cut in half.

BC = 3 cm, AD = 5 cm, AB = CD = DE = JK, FG = CE = GH, EF = 2DE, and $\overline{CG} \parallel \overline{DF} \parallel \overline{JK}$



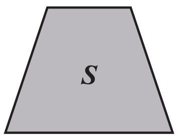
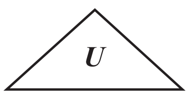
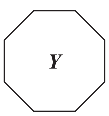
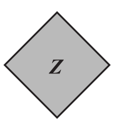
Pattern B is created using shapes Y and Z. To completely fill the rectangle shown, shape Z is cut in half.



1. List at least three types of polygons that can be identified in Pattern A or Pattern B.

2. Refer to Pattern A or Pattern B to complete the following.
   a. Quadrilateral T' is a translation of quadrilateral T __________ centimeters to the __________.
   b. Trapezoid R' is a translation of the trapezoid R _________ centimeters to the __________.
   c. Draw a line on Pattern A which is the line of reflection of trapezoid S to trapezoid S'.
   d. Label triangle U', the reflection over the line drawn in (c).
   e. Draw a line on Pattern B which creates a reflection of quadrilateral Z to quadrilateral Z'.

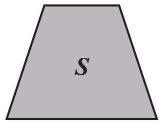
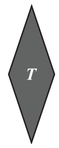
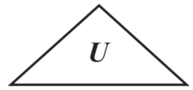
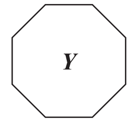
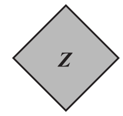
3. Complete the missing information for the table using patterns A and B.

Tile	Area of each tile
	
	
	
	
	$4\frac{5}{7}\text{cm}^2$
	1cm^2

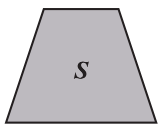
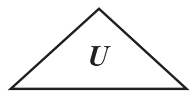
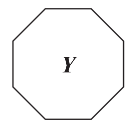
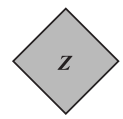
4. Verify that the area of the scale model is 80 square centimeters for each tiling by computing the sum of the areas of the tiles composing each pattern.
   Pattern A:

   Pattern B:

5. The floor plan from the architect shows that the kitchen floor plan is 17.5 cm by 32 cm which is an area of ___________ square centimeters. Determine how many whole tiles will need to be purchased for each pattern to tile the kitchen. The tiles will be cut for the edge pieces.

Figure						
Number of Tiles Needed						

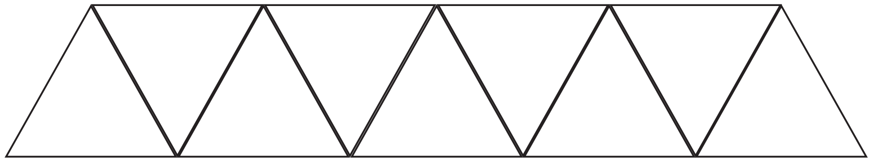
6. The table includes a price for each tile. Determine the total cost to tile the entire kitchen for each floor plan.

Tile						
Cost of each tile	$3.89	$2.99	$2.49	$1.79	$6.29	$2.99

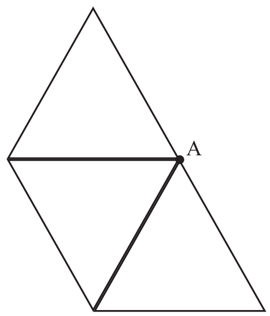
Pattern A: Pattern B:

7. Which tile pattern should the McAlister's choose to stay within their budget?

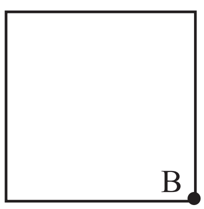
Mrs. McAlister attends an open house for a model home in the area and notices that the kitchen floor is tiled using a tessellation. Tessellations are made by translating, rotating, and reflecting regular polygons so that the polygons are joined vertex to vertex with no gaps. The figure shows one method for tessellating equilateral triangles.

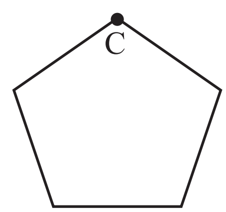


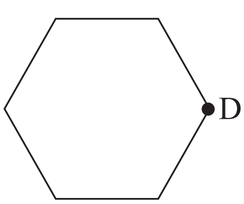
8. Another method for tessellating an equilateral triangle is to rotate the triangle about a point. In the diagram, three congruent equilateral triangles have been joined at point A. Make a sketch that shows the maximum number of congruent equilateral triangles that can be joined at that one vertex without overlap.

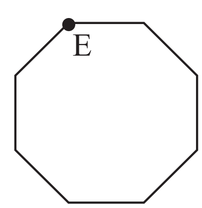


9. Each of the following polygons is regular. Using point B as a vertex, sketch as many congruent squares as possible that can be joined without overlap. Repeat this process for each of the other given polygons. Which regular polygons completely fill the space at the labeled vertex with no spaces and no overlaps?









10. Considering the polygons in questions 8 and 9, complete the table. (Note: The sum of the interior angles of any polygon can be determined by the number of triangles that are formed by drawing all possible diagonals from one specific vertex.)

Number of sides	**Specific name of the regular polygon**	**# of triangles, one vertex connected, times 180°**	**Sum of the interior angles of the given polygon**	**Degree measure of an interior angle of the given polygon**	**Number of congruent polygons that can be joined at the labeled vertex**
3	Equilateral triangle	$(1)(180^\circ)$	180°	60°	6
4					
5					
6					
8					

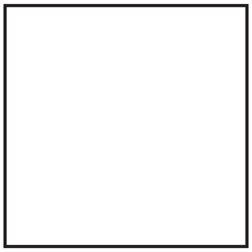
11. Considering the polygons in questions 8 and 9, which of the polygons listed tessellated? Using the drawings and the table, state a rule that explains which regular polygons will and which will not tessellate.

12. After seeing the tile floors in some of new homes in the area, Mrs. McAlister decides that the tile patterns from the architect (Pattern A and Pattern B) are too elaborate for her kitchen floor. She decides she would rather have a floor that uses a basic tessellation pattern with only squares or equilateral triangles. She sees in the newspaper that the local flooring store is having a sale on all floor tiles and wants to see what the price difference will be for each tessellation pattern. Both types of tiles have an original price of $6.99. Square tiles are 20% off and equilateral triangle tiles are 50% off the original price. Mrs. McAlister determines that if she chooses to use squares only, she will need 140 tiles. If she uses equilateral triangles only, she would need 220 tiles. Determine the total cost for each option.

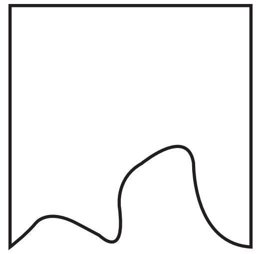
13. Mrs. McAlister decides to buy the triangles to tile her kitchen floor. Will this decision result in her being over budget or under budget? By how much?

14. Mrs. McAlister is considering putting tile in the entry way and wants you to design it. Create a tiling with at least three different polygons or a tessellation of your choosing in the space provided for Mrs. McAlister to consider.

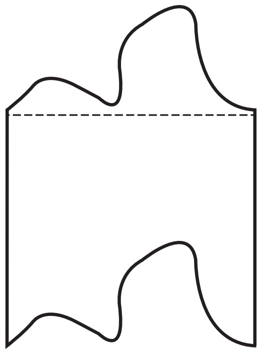
15. A tessellation can become a work of art when it is formed by regular shapes that have been creatively altered so that the copies of the figures continue to fit together without gaps or overlaps. One way to begin is to use a regular polygon. Begin with a square that has an area of 9 in^2. Follow the specific steps to create a tessellating design then translate, slide the shape side to side or up and down with no change to size or shape, the figure to cover a space at least 8.5 inches by 11 inches but no more than 12.5 inches by 15 inches with copies of your tessellation.



To make a tessellation based on a square, cut any irregular shape from the bottom of the square, as shown on the right.

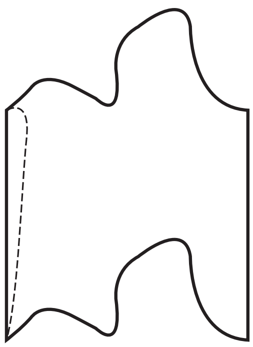


Translate the paper that you cut from the bottom of the square and tape to the top of the square as shown.

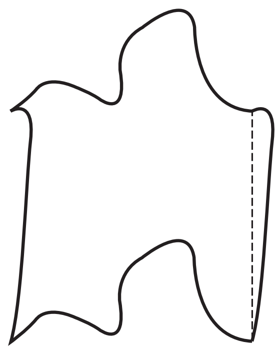


The shape on the left will tessellate. You can stop at this step and use this figure to create your tessellation. What does this shape look like to you? Is it a flower, person, or a ghost?

You can also cut a slice from the left side of the shape as shown here.

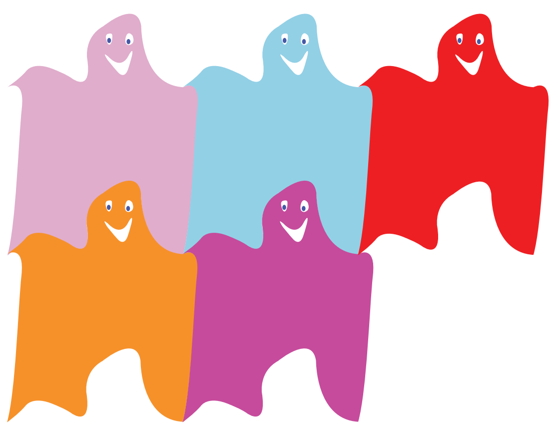


Translate the slice and tape to the right side of the shape.



Add features and color.

Now you have your figure to tessellate. Trace your pattern to form your tessellation. Make sure you cover the space without leaving any gaps or overlaps.



16. Determine the total area covered by the tessellation. Explain your reasoning.

17. Look at the shape you used to create your tessellation in question 15. You may be surprised to see other shapes "appear" when you rotate the figure. This figure that looks like a man's face is almost the same one used for the ghosts. It has been rotated 90° and has an extra cut-out from the left for a mouth. The extra cut-out has then been translated to the right to create more hair.

Using your figure from question 15, apply a similar process (a rotation and an additional cut-out) to create a second figure that will tessellate. Draw a tessellation using four additional copies around your new figure.