Area and Volume Assessment Activity

About this Lesson

Typical lessons on area and volume include questions where students are given either a labeled figure or values for the dimensions and asked to calculate the area or volume by substituting values into a known formula. In previous LTF lessons, students were asked to graph linear equations on a coordinate grid, determine the shape of the figure, and apply the correct formula to determine the area. To extend this concept, students are asked to revolve the 2-dimensional figure about a horizontal or vertical axis to create a 3-dimensional figure and then to determine the figure's volume.

This assessment activity is designed to be used after students have been introduced to creating and determining the areas and volumes of two- and three-dimensional figures on a coordinate plane. The activity begins with multiple choice questions which are similar to those in previous LTF lessons, but then extends the concept to include determining the equation of the line that will create a figure with a given area or volume. The free response portion of the activity requires the students to apply previous learning to a more complex situation. Applying knowledge to a new situation provides students with an opportunity to form a progression of increasing knowledge, skill, and sophistication to promote the sense-making that fuels mastery.

Prior to the lesson, students should be able to determine the equation of a line from two points, use formulas for areas of triangles and rectangles, apply formulas for volumes of cylinders and cones, and create a boxplot.

This lesson is connected to Module 2 – Areas and Volumes and Module 4 – Graphical Displays.

Objectives

Students will

- determine the equation of a line from two points.
- graph linear equations.
- determine the area of triangles and rectangles on a coordinate plane.
- draw and describe the solid formed by revolving the plane figure about a vertical or horizontal line.
- calculate the volume of the solid of revolution.

Level

Algebra 1

Common Core State Standards for Mathematical Content

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (★) at the end of a specific standard indicates that the high school standard is connected to modeling.

Explicitly addressed in this lesson

Code	Standard	Level of Thinking	Depth of Knowledge
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Analyze	III
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★	Analyze	III
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	Analyze	III
F-IF.7a	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph linear and quadratic functions and show intercepts, maxima, and minima.★	Apply	II

Common Core State Standards for Mathematical Practice

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. LTF incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

Implicitly addressed in this lesson

Code	Standard
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
6	Attend to precision.

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LTF Content Progression Chart

In the spirit of LTF's goal to connect mathematics across grade levels, the Content Progression Chart demonstrates how specific skills build and develop from sixth grade through pre-calculus. Each column, under a grade level or course heading, lists the concepts and skills that students in that grade or course should master. Each row illustrates how a specific skill is developed as students advance through their mathematics courses.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure. (6.G.1, 6.G.3, 6.EE.2c, 6.NS.6c, 6.NS.8)	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure. (7.G.6, 6.G.3, 6.EE.2c, 6.NS.6c, 6.NS.8)	Calculate the area of a triangle, rectangle, trapezoid, or composite of these three figures formed by linear equations and/or determine the equations of the lines that bound the figure. (G-GPE.7, A-REI.12, F.IF.7a)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations or equations of circles and/or determine the equations of the lines and circles that bound the figure. (G-GPE.7, A-REI.12, F.IF.7a)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure. (G-GPE.7, A-REI.12, F.IF.7a)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure. (G-GPE.7, A-REI.12, F.IF.7a)
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines. (6.G.3, 6.EE.2c, 6.NS.6c, 6.NS.8, 8.G.9)	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines. (6.G.3, 6.EE.2c, 6.NS.6c, 6.NS.8, 8.G.9)	Given the equations of lines (at least one of which is horizontal or vertical) that bound a triangular, rectangular, or trapezoidal region, calculate the surface area and/or volume of the region formed by revolving the region about the line that is horizontal or vertical. (8.G.9, G-GMD.3, G-GMD.4, F-IF.7a)	Given the equations of lines or circles that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (G-GMD.3, G-GMD.4, F-IF.7a)	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (G-GMD.3, G-GMD.4, F-IF.7a)	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (G-GMD.3, G-GMD.4, F-IF.7a)

Connection to AP*

AP Calculus Topic: Areas and Volumes

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Materials and Resources

- Student Activity pages

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Assessments

The following types of formative assessments are embedded in this lesson:

- Students engage in independent practice.
- Students apply knowledge to a new situation.

The following additional assessments are located on the LTF website:

- Areas and Volumes – Algebra 1 Free Response Questions
- Areas and Volumes – Algebra 1 Multiple Choice Questions

Teaching Suggestions

This lesson can be used as a collaborative learning project or as a formative assessment after students have had experience with questions such as those included in the following LTF lessons: Solids of Revolution (MG); Solving Systems of Linear Equations (A1); Volumes of Revolution (GE). The free response question appeared on the 2011 Algebra 1 LTF Posttest. It was part of a series of questions that showed the vertical alignment of the concept from sixth grade through pre-calculus. Student samples for each grade level can be found on the LTF website.

Modality

LTF emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using these representations to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

Answers

1. Rationales:
   A. Correct. Student uses a base of 6 units and a height of 4 units.
   B. Student uses $x = 3$, the *x*-axis, and $y = -2(x-6)+1$. Student uses a base of 3.5 units and a height of 7 units.
   C. Student uses a base of 6 units and a height of 7 units (counts the height from the *x*-axis rather than from $y = 3$).
   D. Student uses $x = 3$, the *x*-axis, and $y = x + 4$. Student uses a base of 7 units and a height of
   7 units.
   E. Student uses the *x*-axis rather than $y = 3$. Student uses a base of 10.5 units and a height of
   7 units.

2. Rationales
   I. True. Student graphs the line and determines that a triangle in the first quadrant has a base of 10 units on the *y*-axis and a height of 2 units, giving an area of 10 square units.
   II. True. Student graphs the line and determines that the obtuse triangle in the first and fourth quadrants has a base of 10 units on the *y*-axis and a height of 2 units, giving an area of 10 square units.
   III. True. Student graphs the line and determines that the obtuse triangle in the third quadrant has a base of 5 units on the *y*-axis and a height of 4 units, giving an area of 10 square units.
   A. Student selects the first equation that creates a triangle with area of 10 square units.
   B. Student only considers one equation that creates a triangle with an area of 10 square units.
   C. Student only considers one equation that creates a triangle with an area of 10 square units.
   D. Student selects two of the three equations that create a triangle with an area of
   10 square units.
   E. Correct. Student recognizes that all three equations create triangles with areas of
   10 square units.

3. Rationales
   A. Student revolves the figure about the *x*-axis.
   B. Student uses 6 units for the height and 3 units for the radius, or student revolves the figure about the *x*-axis, and then divides by 2 rather than 3.
   C. Correct. Student uses a radius of 6 units and a height of 3 units to determine the volume of the cone.
   D. Student divides by 2 rather than 3 in the formula for the volume of a cone. Alternatively, the student revolves the figure about the *x*-axis or uses 3 units for the radius and 6 units for the height and does not divide by 3.
   E. Student does not divide by 3.

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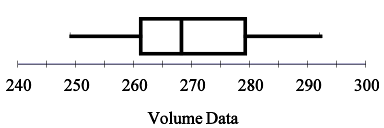
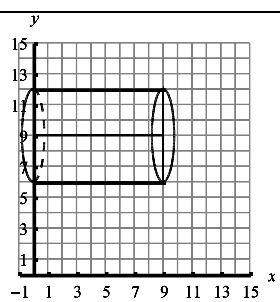
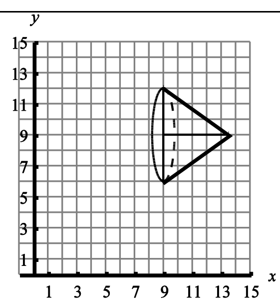
4. Rationales:
   A. Student solves $\pi r^2(6) = 54\pi$ and then uses the *r* value as the area.
   B. Student uses the *h* value as the area.
   C. Student squares the height, 6 units, rather than the radius.
   D. Correct. Student solves $\pi r^2(6) = 54\pi$ to determine that the radius is 3 units and multiplies by the height, which is 6 units, to determine the area of the rectangle.
   E. Student multiplies the radius by 2 rather than squaring the radius.

5. Rubric provided on the following page.

(a) $h(x)=\frac{2}{3}x$ $9=\frac{2}{3}x$ $x=\frac{27}{2}$ Area $=\frac{1}{2}(3)\left(\frac{9}{2}\right)=\frac{27}{4}$ square units	(a) 3 pts: 1 pt: correct $h(x)$ equation 1 pt: correct x-value 1 pt: correct area based on student's $(x-9)$ value
(b) Revolving region R about $y=9$: $\pi(3^2)(9)=81\pi$ cubic units	(b) 1 pt: 1 pt: correct volume for cylinder formed by revolving region R
(c) Revolving region S about $y=9$: $\frac{\pi}{3}(3^2)\left(\frac{9}{2}\right)=\frac{27\pi}{2}$ cubic units	(c) 1 pt: 1 pt: correct volume for cone formed by revolving region S based on student's $x-9$ value or height in part (a)
(d) Revolving region Z about $y=9$: $\frac{\pi}{3}(9^2)\left(\frac{27}{2}\right)=\frac{729\pi}{2}$ cubic units	(d) 1 pt: 1 pt: correct volume for cone formed by revolving region Z based on student's x value or sum of heights used for regions R and S
(e)  No, because the boxplot does not display all of the actual data points so it is unknown whether a data point exists between 268 and 270.	(e) 2 pts: 1 pt: correct boxplot 1 pt: correct answer with reason
(s)  	(s) 1 pt: 1 pt: correct sketch of part (b) or (c)

T E A C H E R

Student Sample (earned 9 pts of 9 pts)
Additional student samples available at www.ltftraining.org

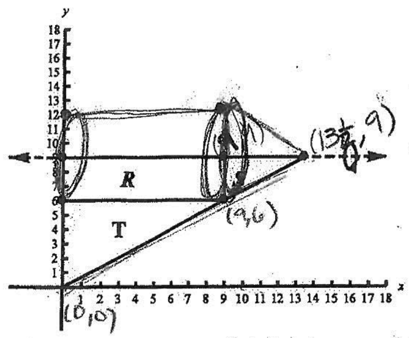
LTF Posttest Part B

Algebra 1

Sample A

In the figure provided:

- Region R is a rectangle bounded by the lines $y = 6$, $x = 0$, $y = 9$, and $x = 9$.
- Region S is a triangle bounded by the lines $x = 9$, $y = 9$, and a line, $h(x)$, containing the points $(0, 0)$ and $(9, 6)$.
- Region T is a triangle bounded by $y = 6$, $x = 0$, and the line, $h(x)$.
- Region Z is formed by grouping regions R, S, and T.



(a) What is the equation of the line, $h(x)$, containing the points $(0, 0)$ and $(9, 6)$?

$m = \frac{y_2 - y_1}{x_2 - x_1}$ $m = \frac{6-0}{9-0} = \frac{2}{3}$ $y = ax + b$ $6 = \frac{2}{3}(\frac{9}{1}) + b$ $6 = 6 + b$ $y = \frac{2}{3}x$

What is the value of x when $h(x) = 9$? Show the work that leads to the answer. Using this information, identify the point on region S by labeling the coordinates.

(13½, 9)

$y = \frac{2}{3}x$

$\frac{3}{2} \cdot \frac{9}{1} = \frac{2}{3}x \cdot \frac{3}{2}$

$x = \frac{27}{2} = 13\frac{1}{2}$

Identify the other two vertices of region S by labeling their coordinates. What is the area of region S? Show the work that leads to the answer.

$9 + 4\frac{1}{2} = 13\frac{1}{2}$

(9,9) 4½ (13½, 9) 3 (9,6)

$A = \frac{1}{2}bh$

$A = \frac{1}{2}(4\frac{1}{2})(3)$

$A = \frac{1}{2}(13.5)$

$A = 6.75\ un^2$

$13.50 \div 2 = 6.75$

$1.5 \times 3 = 3.5$

(b) When region R is revolved about the line $y = 9$, a cylinder is formed. Sketch the cylinder on the graph by reflecting the rectangle across the line $y = 9$. Connect the corresponding outside vertices of the combined regions with oval-like shapes.

What is the volume of the cylinder? Show the work that leads to the answer. Leave π in the answer.

$V = Bh$

$V = \pi R^2 h$

$V = \pi(3^2)(9)$

$V = \pi(9)(9)$

$V = 81\pi\ un^3$

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GO ON TO THE NEXT PAGE

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Posttest | Part B

Algebra 1

Sample A

(c) When region S is revolved about the line $y=9$, a cone is formed. On the graph provided on the previous page, sketch the cone in the correct position.

What is the volume of the cone in terms of π? Show the work that leads to the answer.

($V=\frac{1}{3}Bh$, where B is the area of the base and h is the height of the cone)

$V=\frac{1}{3}Bh$

$V=\frac{1}{3}(\pi r^2)(h)$

$V=\frac{1}{3}(\pi)(3^2)(4\frac{1}{2})$

$V=\frac{1}{3}(\pi)(9)(\frac{9}{2})$

$V=(\pi)(9)(\frac{3}{2})$ $V=\frac{27}{2}\pi$ un^3

$\frac{9^3}{2}\times\frac{1}{3}$ $\frac{3}{2}$

(d) When region Z is revolved about the line $y=9$, a cone with a radius of 9 is formed. What is the volume of the cone in terms of π? Show the work that leads to the answer.

$V=\frac{1}{3}Bh$

$V=\frac{1}{3}(\pi r^2)(h)$

$V=\frac{1}{3}(\pi)(9^2)(13\frac{1}{2})$

$V=\frac{1}{3}(\pi)(81)(13\frac{1}{2})$

$V=\frac{1}{3}(\pi)(81)(\frac{27}{2})$

$V=(\pi)(\frac{81}{1})(\frac{9}{2})$

$V=\pi(\frac{729}{2})$ un^3

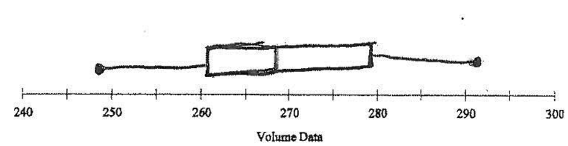
or

$V=364.5\pi$ un^3

(e) When region T is revolved about the line $y=9$, a frustum with a cylinder removed is formed that has a volume of 270π cubic units. Two hundred students measured the dimensions of a wooden model of the solid and calculated the volume. They divided their calculations by π and collected the data. The five-number summary for the data is

{min = 249, Q1 = 261, med = 268, Q3 = 279, and max = 292}

Create a boxplot of the data.



Is the statement: "Based on the boxplot, at least one student calculated a volume between the median and the actual volume" correct? Justify the answer.

no, there might not be any data points between 268 and 270

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Check this box if you prefer your work not be used in relation to teacher training.

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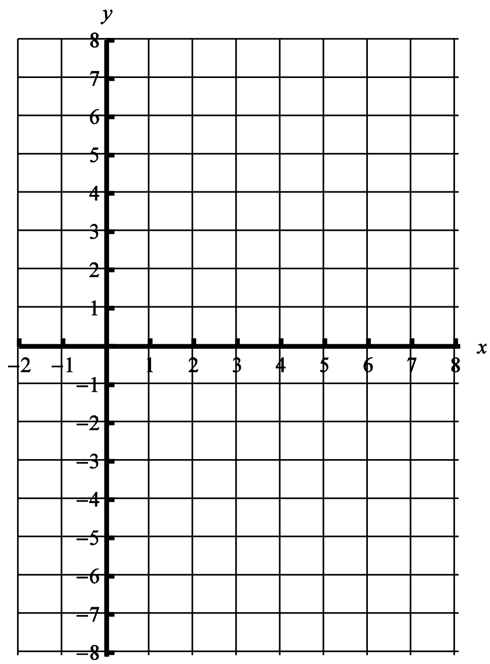
Area and Volume Assessment Activity

1. A triangular region is bounded by the lines $y = 3$, $y = x + 4$, and $y = -2(x - 6) + 1$.



What is the area of the triangular region?

A. 12 square units

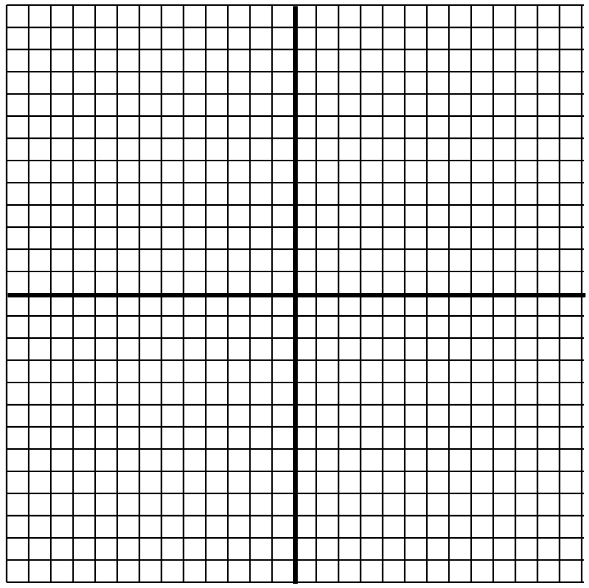
B. $12\frac{1}{4}$ square units

C. 21 square units

D. $24\frac{1}{2}$ square units

E. $36\frac{3}{4}$ square units

2. A triangle whose area is 10 square units is formed by the intersection of three lines. The equations of two of the lines are $x = 0$ and $y = 3x$.

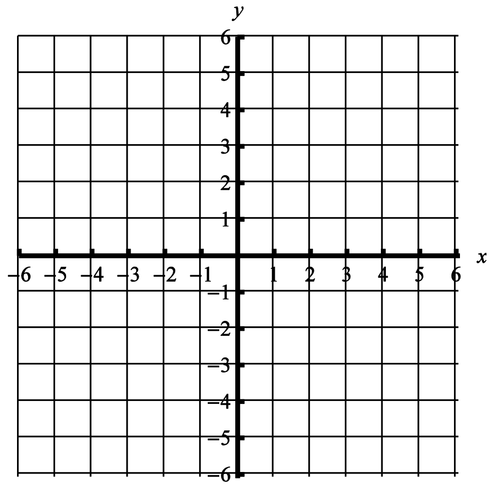


Which of the following could be the equation of a line containing the third side?

I. $y = -2x + 10$
II. $y = 8(x-1) - 2$
III. $y = \frac{7}{4}(x+4) - 12$

A. I only
B. II only
C. III only
D. II and III only
E. I, II, and III

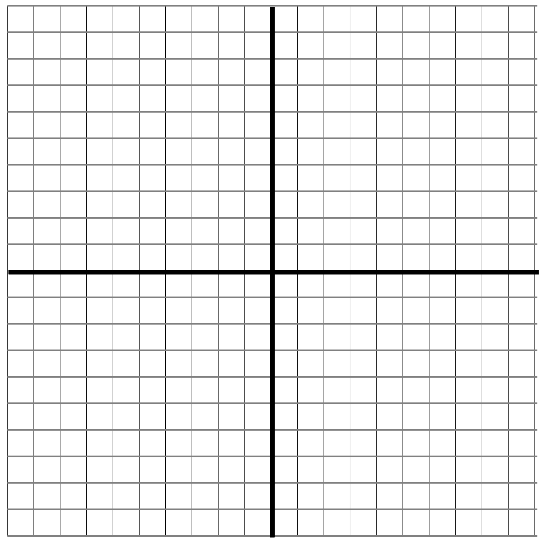
3. A triangular region is bounded by the lines $x = 0$, $y = 0$, and $3x + 6y = 18$.



What is the volume of the cone that is formed when the triangular region is revolved about the y-axis?

A. 18π cubic units
B. 27π cubic units
C. 36π cubic units
D. 54π cubic units
E. 108π cubic units

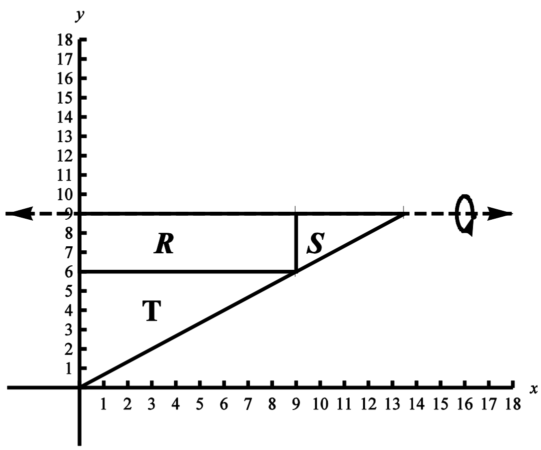
4. The volume of a cylinder that is created by revolving a rectangle about the *y*-axis is 54π cubic units. The rectangle is formed by the intersection of four lines. The equations of three of the lines are: $x = 0$, $y = 0$, and $y = 6$.



Which of the following is the area of the rectangle that is revolved to form the cylinder?

A. 3 square units
B. 6 square units
C. 9 square units
D. 18 square units
E. 27 square units

5. In the figure provided:
   - Region R is a rectangle bounded by the lines $y=6$, $x=0$, $y=9$, and $x=9$.
   - Region S is a triangle bounded by the lines $x=9$, $y=9$, and a line, $h(x)$, containing the points $(0, 0)$ and $(9, 6)$.
   - Region T is a triangle bounded by $y=6$, $x=0$, and the line, $h(x)$.
   - Region Z is formed by grouping regions R, S, and T.



(a) What is the equation of the line, $h(x)$, containing the points $(0, 0)$ and $(9, 6)$?

What is the value of x when $h(x)=9$? Show the work that leads to the answer. Using this information, identify the point on region S by labeling the coordinates.

Identify the other two vertices of region S by labeling their coordinates. What is the area of region S? Show the work that leads to the answer.

(b) When region R is revolved about the line $y=9$, a cylinder is formed. Sketch the cylinder on the graph by reflecting the rectangle across the line $y=9$. Connect the corresponding outside vertices of the combined regions with oval-like shapes.

What is the volume of the cylinder? Show the work that leads to the answer. Leave π in the answer.

(c) When region S is revolved about the line $y = 9$, a cone is formed. On the graph provided on the previous page, sketch the cone in the correct position.

What is the volume of the cone in terms of π? Show the work that leads to the answer.

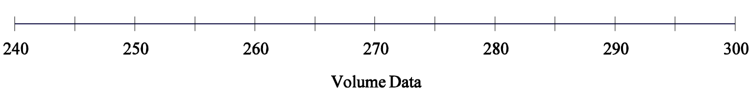
($V = \frac{1}{3}Bh$, where B is the area of the base and h is the height of the cone)

(d) When region Z is revolved about the line $y = 9$, a cone with a radius of 9 is formed. What is the volume of the cone in terms of π? Show the work that leads to the answer.

(e) When region T is revolved about the line $y = 9$, a frustum with a cylinder removed is formed that has a volume of 270π cubic units. Two hundred students measured the dimensions of a wooden model of the solid and calculated the volume. They divided their calculations by π and collected the data. The five-number summary for the data is

$$\{\text{min} = 249,\ Q_1 = 261,\ \text{med} = 268,\ Q_3 = 279,\ \text{and max} = 292\}$$

Create a boxplot of the data.



Is the statement: “Based on the boxplot, at least one student calculated a volume between the median and the actual volume” correct? Justify the answer.

