Volumes of Revolution

About this Lesson

This lesson provides students with a physical method to visualize 3-dimensional solids and a specific procedure to sketch a solid of revolution. Students will determine the area of two-dimensional figures created on a coordinate plane. In addition, students will determine the volume of three-dimensional figures created by revolving the region on the coordinate plane about a horizontal or vertical line.

Prior to the lesson, students should be able to plot points on a coordinate plane, calculate areas of plane figures, and calculate volumes of solids.

This lesson is included in Module 2: Areas and Volumes.

Objective

Students will

- plot the points for a plane figure on a coordinate plane.
- determine the area of that plane figure.
- draw and describe the solid formed by revolving the plane figure about a vertical or horizontal line.
- calculate the volume of the solid.
- compare the volumes determined when revolving about different axes.

Level

Geometry

Common Core State Standards for Mathematical Content

This lesson addresses the following Common Core Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (★) at the end of a specific standard indicates that the high school standard is connected to modeling.

Explicitly addressed in this lesson

Code	Standard	Level of Thinking	Depth of Knowledge
G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★	Analyze	III
A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	Analyze	III

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Code	Standard	Level of Thinking	Depth of Knowledge
G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	Analyze	III
G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.⋆	Analyze	III

Common Core State Standards for Mathematical Practice
These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. LTF incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connection across grade levels. This lesson allows teachers to address the following Common Core Standards for Mathematical Practice.

Implicitly addressed in this lesson

Code	Standard
1	Make sense of problems and persevere in solving them.
2	Reason abstractly and quantitatively.
3	Construct viable arguments and critique the reasoning of others.
4	Model with mathematics.
6	Attend to precision.

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LTF Content Progression Chart

In the spirit of LTF's goal to connect mathematics across grade levels, the Content Progression Chart demonstrates how specific skills build and develop from sixth grade through pre-calculus. Each column, under a grade level or course heading, lists the concepts and skills that students in that grade or course should master. Each row illustrates how a specific skill is developed as students advance through their mathematics courses.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure. (200_06.AV_B02)	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure. (200_07.AV_B02)	Calculate the area of a triangle, rectangle, trapezoid, or composite of these three figures formed by linear equations and/or determine the equations of the lines that bound the figure. (200_A1.AV_B02)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations or equations of circles and/or determine the equations of the lines and circles that bound the figure. (200_GE.AV_B02)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure. (200_A2.AV_B02)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure. (200_PC.AV_B02)
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines. (200_06.AV_B03)	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines. (200_07.AV_B03)	Given the equations of lines (at least one of which is horizontal or vertical) that bound a triangular, rectangular, or trapezoidal region, calculate the surface area and/or volume of the region formed by revolving the region about the line that is horizontal or vertical. (200_A1.AV_B03)	Given the equations of lines or circles that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (200_GE.AV_B03)	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (200_A2.AV_B03)	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the region formed by revolving the region about a horizontal or vertical line. (200_PC.AV_B03)

Connections to AP*

AP Calculus Topics: Areas and Volumes

Materials

Student Activity pages

T E A C H E R

Assessment

The following types of formative assessments are embedded in this lesson:

- Students engage in independent practice.
- Students construct and use manipulatives.
- Students apply knowledge to a new situation.

The following additional assessments are located on the LTF website:

- Areas and Volumes – Geometry Free Response Questions
- Areas and Volumes – Geometry Multiple Choice Questions

Teaching Suggestions

The concept of revolving a region about an axis is fundamental to integral calculus. This lesson includes calculating perimeter and area of a planar region and then the volume generated by rotating the region around the x-axis or the y-axis.

To help students visualize the solid generated by revolving the figure about an axis, have them glue or tape a triangle onto a stick or dowel and then rotate the triangle horizontally and vertically between their hands so the students can “see” the cone that will be generated.

An extension of this lesson is to bring 3D objects to class and having the students draw cross-sections. Being able to visualize solids is a valuable tool for calculus.

When drawing a sketch of a solid revolution, use the following procedure:

- Draw the boundaries.
- Shade the region to be revolved.
- Draw the reflection (mirror image) of the region about the axis of rotation.
- Connect significant points and their reflections with ellipses.

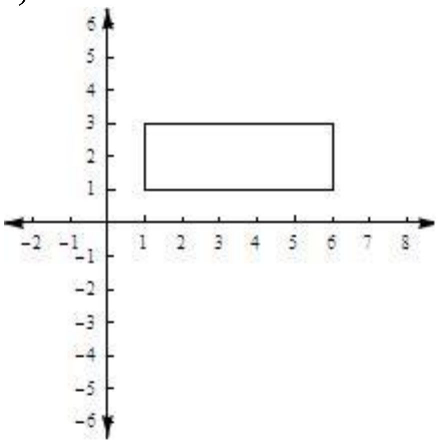
Modality

LTF emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using these representations to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

Answers

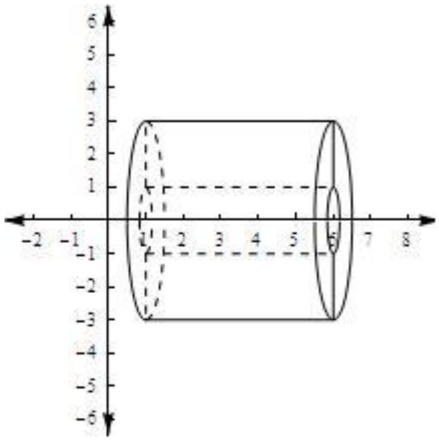
1. a)



b) 14 units

c) 10 sq. units

d)



e) a cylinder with a cylinder removed

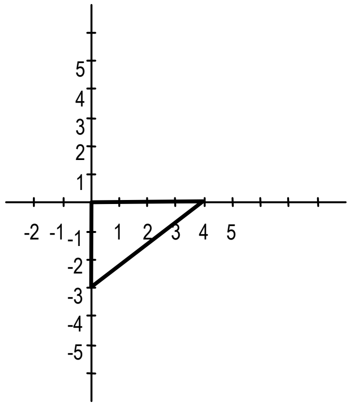
f) $V = \left(\pi R^2 h - \pi r^2 h\right)$

$= \pi h\left(R^2 - r^2\right)$

$= \pi(5)\left(3^2 - 1^2\right)$

$= 40\pi$ cu. units

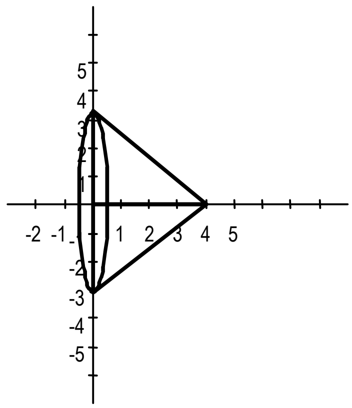
2. a)



b) 12 units

c) 6 sq. units

d)



e) cone

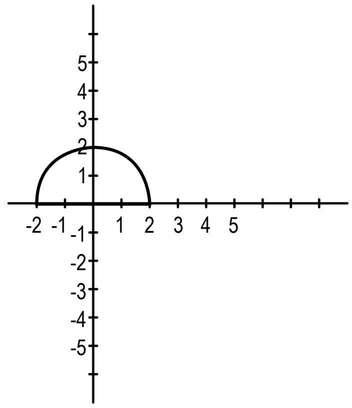
f) 12π cu. units

g) 24π sq. units (remember the base)

h) The volume is 16π cu. units, so the *y*-axis rotation has greater volume. The surface area, 36π sq. units, is greater in the *y*-axis rotation as well.

i) Region bounded by $y = -\frac{3}{4}x + 3$, $y = 0$ and $x = 0$.

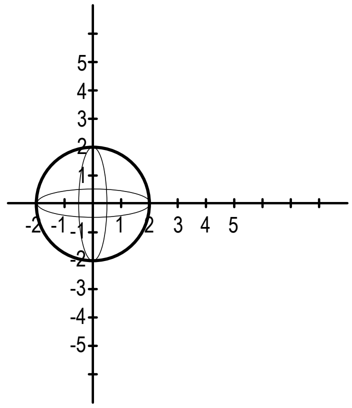
3. a)



b) $(2\pi + 4)$ units

c) 2π sq. units

d)



e) sphere

g) $\dfrac{32\pi}{3}$ cu. units

h) 16π sq. units

i) The geometric solid would be the top half of the sphere, called a hemisphere. The volume would be half the volume of the sphere, or $\dfrac{16\pi}{3}$ cu. units.

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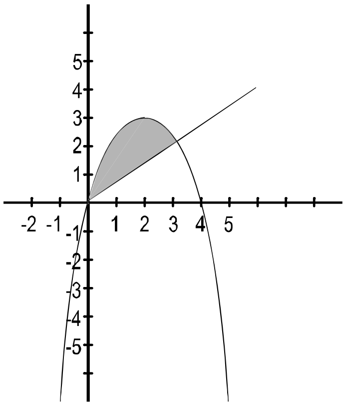
4. a)



5
4
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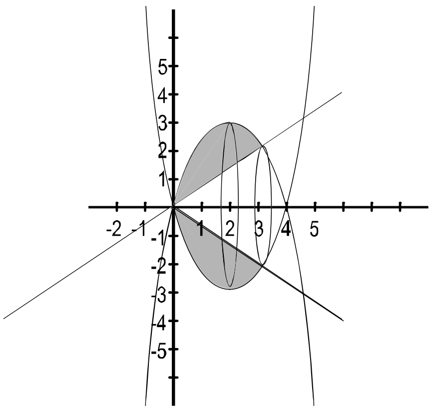
-2 -1
1 2 3 4 5
-1
-2
-3
-4
-5

b)



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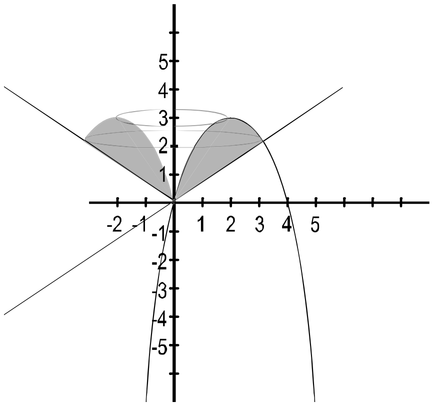
-5

c)



5
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1 2 3 4 5
-1
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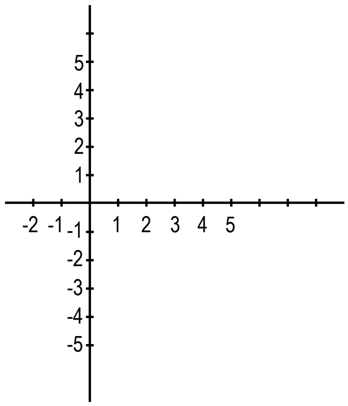
-5

Volumes of Revolution

1. Sketch the region bounded by the lines $y = 3$, $y = 1$, $x = 1$, $x = 6$.

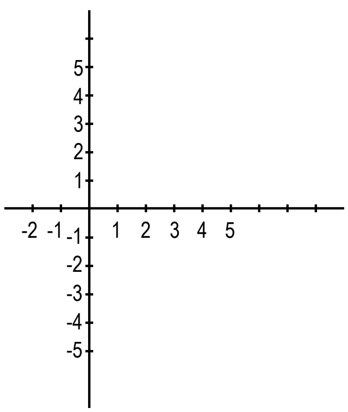
c)



d) Determine the perimeter of the region.

e) Determine the area of the region.

f) Draw a picture of the region being revolved about the x-axis.

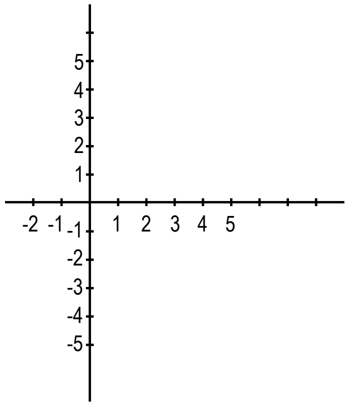


g) Describe the geometric solid formed by revolving the region about the x-axis.

h) Determine the volume of the geometric solid.

2. Sketch the region bounded by the lines $y = \frac{3}{4}x - 3$, $y = 0$, $x = 0$.

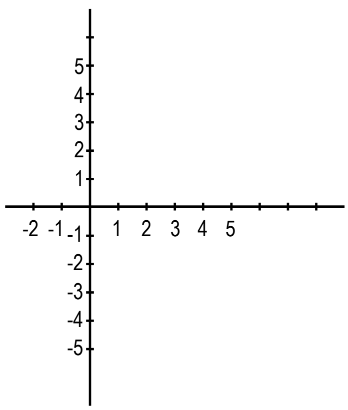
a)



b) Determine the perimeter of the region.

c) Determine the area of the region.

d) Draw a picture of the region being revolved about the *x*-axis.



e) What geometric figure is formed by revolving the region about the *x*-axis?

f) Determine the volume of the geometric solid.

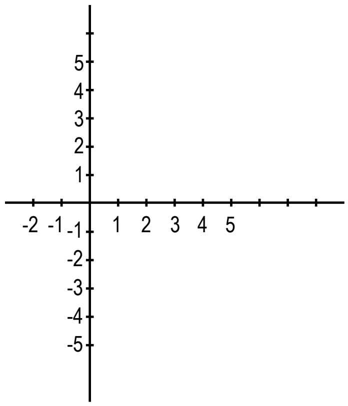
g) Determine the surface area of the geometric solid.

h) If the region were revolved about the *y*-axis, would the volume be greater than, less than, or equal to the volume formed by revolving about the *x*-axis? Justify your answer. Compare the surface areas.

i) Name another region that could be revolved about the *x*-axis to create exactly the same geometric solid.

3. Sketch the region bounded by the curve $y = \sqrt{4 - x^2}$ and the line $y = 0$.

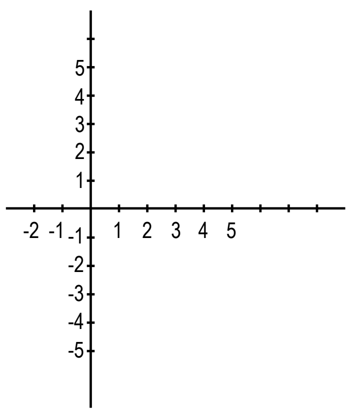
a)



b) Determine the perimeter of the region.

c) Determine the area of the region.

d) Draw a picture of the region being revolve about the *x*-axis.



e) What geometric figure is formed by revolving the region about the *x*-axis?

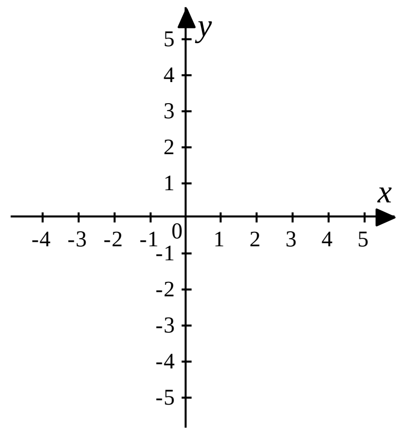
f) Determine the volume of the geometric solid.

g) Determine the surface area of the geometric solid.

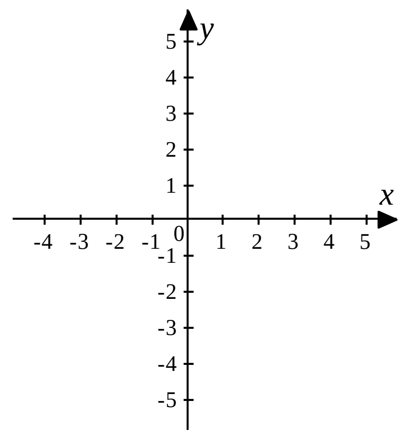
h) If the region were rotated about the *y*-axis, would the volume be greater than, less than, or equal to the volume formed by revolving about the *x*-axis? Justify your answer.

4. A region is bounded by the graphs
   $y = 3 - \frac{3}{4}(x-2)^2$ and $y = \frac{2}{3}x$.

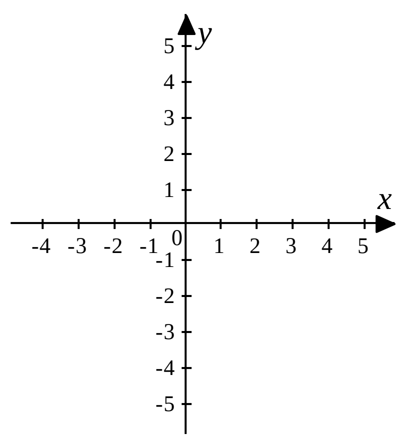
   a) Draw a picture of the region.



   b) Draw a picture of the region rotated around the *x*-axis.



   c) Draw a picture of the region rotated around the *y*-axis.



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- Draw the boundaries.
- Shade the region to be revolved.
- Draw the reflection (mirror image) of the region across the axis of revolution.
- Connect significant points and their reflections with ellipses.

