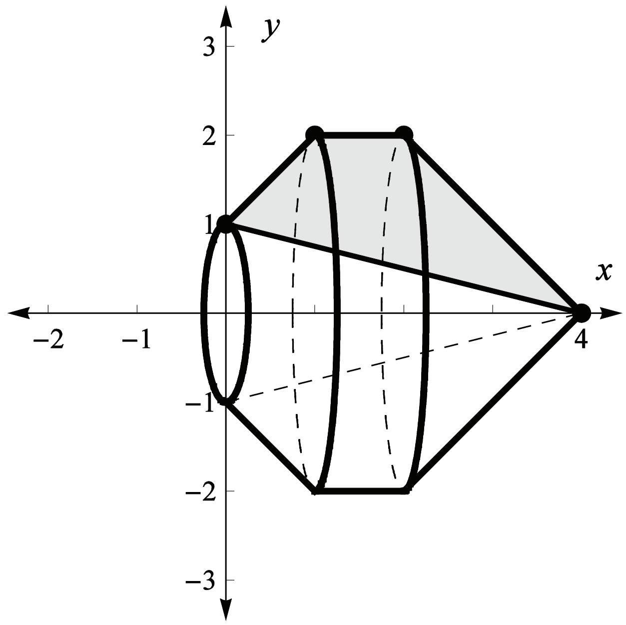


Using Linear Equations to Define Geometric Solids

ABOUT THIS LESSON

In the first part of the lesson, students plot ordered pairs, graph linear functions defined by a variety of representations, and then create two-dimensional figures using the lines as boundaries. Students are asked to calculate the perimeters and the areas of the bounded regions and then to describe and sketch the three dimensional figures formed when the planar regions are revolved about a horizontal or vertical line. Students must determine the dimensions of the solids based on their understanding of how the shapes are created and then calculate the volumes. As students compare the volumes when the same planar region is revolved about different lines, they develop a deeper conceptual understanding about how the change in the radius impacts the overall volume of a solid. The activity, which includes an interactive computer demonstration and a real-world model of a custom bowling ball, provides an engaging setting for students to practice and apply their skills with graphing linear functions, as well as calculating areas and volumes.

The lesson is designed to enhance student understanding of the Common Core State Standards by developing coherence and connections among a variety of mathematical concepts, skills, and practices. Each question in the lesson requires students to incorporate several of the targeted and reinforced/applied standards in new or non-standard situations, including graphing linear equations, recognizing two-dimensional bounded regions and the three-dimensional figures they sweep out by rotation, as well as calculating perimeters, areas, and volumes.

LEVEL

Geometry within a unit on volume applications

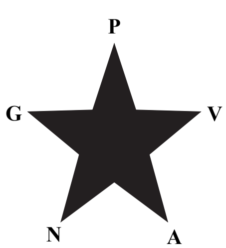
MODULE/CONNECTION TO AP*

Area and Volume

**Advanced Placement and AP are registered trademarks of the College Entrance Examination Board. The College Board was not involved in the production of this product.*

MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using these representations to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

TEACHER PAGES

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

OBJECTIVES

Students will

- write the equations of the lines that define bounded regions.
- graph linear equations to define bounded regions.
- calculate the areas and perimeters of the regions.
- revolve the planar region about horizontal and vertical lines to create solids.
- determine the volumes of geometric solids that result from revolving the regions about horizontal and vertical lines.
- use technology to create and revolve planar figures about horizontal and vertical lines.
- model the conceptual understanding using a real world situation.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill. The star symbol (★) at the end of a specific standard indicates that the high school standard is connected to modeling.

Targeted Standards

G.GMD.4: Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
See questions 1d, 1f, 2d, 3b, 3d-e, 4c, 5b-c, 5e, 6c-e, 7d, 7f, 8a, 8d

G.GPE.7: Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
See questions 1c, 1e, 2b-c, 3c, 4b, 5a, 6c, 7a

G.GMD.3: Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★
See questions 1e, 1f, 2d-e, 3d-e, 4c, 5d-e, 6d-e, 7d, 7f-g, 8c-e

G.MG.1: Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
See questions 8a,8d

Reinforced/Applied Standards

A.CED.2: Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★
See questions 1b, 2a, 3a, 7b

A.REI.6: Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
See questions 2b, 3b, 4b, 6b.

F.IF.7a: Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
(a) Graph linear and quadratic functions and show intercepts, maxima, and minima.
See questions 3b, 4a, 6a

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.1: Make sense of problems and persevere in solving them.
Students graph equations, identify the planar region formed, and use a variety of algebraic techniques to determine the dimensions necessary to calculate the perimeter and area of the region and the volume of the rotated figure.

Students engage in productive struggle as they persist in visualizing the 3-dimensional shapes created and calculating the volumes in question 7.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

MP.2: Reason abstractly and quantitatively.
Students progress from computing volumes of known shapes numerically to computing volumes of non-standard shapes numerically to computing volumes with generic dimensions.

MP.3: Construct viable arguments and critique the reasoning of others.
In question 2, students explain whether revolving a figure about the x- or y-axis produces a solid with greater volume.

In question 5, students determine the relationship between the dimensions of a planar figure that when revolved about the x- or y-axis will produce solids with equal volume.

In question 7, students must persist in explaining how to decompose a non-standard shape to determine how to calculate its volume.

MP.4: Model with mathematics.
In question 8, students use solids of revolution to model the manufacturing process of creating a custom bowling ball.

MP.5: Use appropriate tools strategically.
Students create 3-dimensional models using pencil and paper and computer-generated models to visualize the solids of revolution.

MP.6: Attend to precision.
In question 7d and 7f, students explain how to calculated the volume using accurate compositions and decompositions of other solids.

MP.7: Look for and make use of structure.
Students recognize that composition and decomposition of solids is needed to calculate the volume of the generated solids.

In question 7, students draw auxiliary lines to create a known planar figure and then visualize the solid formed by revolving the planar figure about an axis and compute its volume by decomposing the solid into known three-dimensional figures.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Graph linear equations
- Calculate perimeter and area of planar figures and volumes of cylinders and cones

ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Students summarize a process or procedure.
- Students engage in independent practice.
- Students apply knowledge to a new situation.

The following additional assessments are located on the NMSI website:

- Areas and Volumes – Geometry Free Response Questions
- Areas and Volumes – Geometry Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
- Cut-out of planar region
- Pencil, straw, or skewer
- Tape
- Scientific or graphing calculators
- Graph paper
- Mathematica demonstration to visualize a planar region revolved about an axis is located on the NMSI website with resources for this lesson (a free download of Mathematica Player is required).
- Applets for creating volumes of revolution:
  http://www.math.psu.edu/dlittle/java/calculus/volumewashers.html
  http://www.ies.co.jp/math/java/calc/rotate/rotate.html
  http://demonstrations.wolfram.com/SolidsOfRevolution/

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

TEACHING SUGGESTIONS

Many students have difficulty in visualizing three-dimensional solids. Before beginning this lesson, model the revolutions about different axes by taping a cut-out of one of the planar regions described in the activity to a pencil, straw, or skewer, and then rolling the pencil between your hands to simulate the "sweeping out" of the solid. Ask students to describe the three-dimensional shape that is produced by revolving the figure and to identify the radius and height of the solid. To help students visualize objects formed when a planar shape is revolved about either a horizontal or vertical axis, use the applet designed to illustrate question 7.

The questions in this lesson build from cylinders and cones to figures that are a combination of solids. When drawing a sketch of a solid revolution, have students use the following procedure:

- Draw the boundaries.
- Shade the region to be revolved.
- Draw the reflection (mirror image) of the region across the axis of revolution.
- Connect significant points and their reflections with ellipses (oval shapes) to illustrate the solid nature of the figure.

A group setting is most appropriate for this lesson and encourages discussion of various approaches to answering the questions. The use of cooperative groups will encourage all students, including ELL students, to share information while working together to complete authentic tasks. The use of cooperative groups will also encourage active engagement in the formation of a conceptual base enhanced by investigating multiple modalities within the problem situation. Incorporating group work is one way to give students space to learn academic language while absorbing content. Students who might be reluctant to talk in whole-class discussions can practice using mathematical language and receive feedback in a relatively low-stakes setting during group work. In this lesson, a student who is beginning to learn English can engage by visualizing graphs and determining volumes. With some assistance from a bilingual peer, these students can also write descriptions of the solids and explain their reasoning about the differences in the volumes based on the axis of revolution without compromising or simplifying the mathematical language.

This lesson is not designed to be completed during one class period and can be separated into parts easily. The first part of the assignment would be to complete questions 1 through 4 to assure that students have a firm understanding of how to sketch the region, revolve the figure, and calculate the volume. Although the modality of the initial stem varies in each of the four questions, the teacher could select one of these questions to use as an assessment. Questions 5 and 6 make up the second part of the assignment and include generically defined functions to increase the rigor. For the third assignment, all students could use the Mathematica demonstration to sketch the figure and could complete questions 7a-c and 7e; however, parts (d), (f), and (g) are more appropriate for advanced or gifted learners. Question 8 could be used as a performance-based assessment question.

Additional lessons at this grade level as well as lower grade levels are available on the NMSI website to assist with scaffolding this concept. For example, the middle grades lesson "Solids of Revolution" generates the region by plotting points rather than graphing equations and limits the solids of revolution to those that are relatively easy to visualize. The Algebra 1 lesson, "Solving System of Linear Equations," defines the two-dimensional regions by graphing linear equations but limits the calculations to areas only. The geometry lesson, "Volumes of Revolution" can also be used to develop foundational understanding.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

Suggested modifications for additional scaffolding and extensions include the following:

1-4	Provide models of each solid using paper and straws or computer applets. Students with some learning disabilities may only be able to visualize the solids which do not have other solids removed, such as 1a-e, 2a-e, 3a-d, 4a-c.
5-6	Provide numerical substitutions for the variable and then convert back to variables.
7	Assign parts (a)-(c) and (e) to all students and in addition, assign parts (d), (f), and (g) to gifted learners
8	Bring a bowling ball to class to use as a demonstration. As an extension, ask students to write the equation of the semicircle.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, the Content Progression Chart demonstrates how specific skills build and develop from sixth grade through pre-calculus. Each column, under a grade level or course heading, lists the concepts and skills that students in that grade or course should master. Each row illustrates how a specific skill is developed as students advance through their mathematics courses.

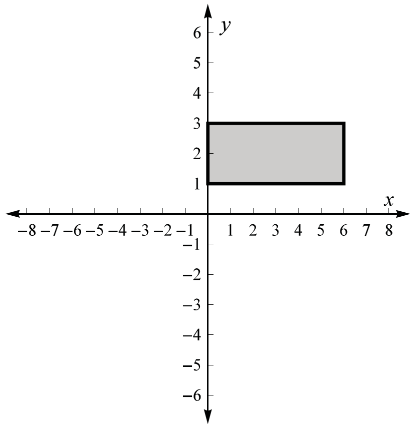
6th Grade Skills/ Objectives	7th Grade Skills/ Objectives	Algebra 1 Skills/ Objectives	Geometry Skills/ Objectives	Algebra 2 Skills/ Objectives	Pre-Calculus Skills/ Objectives
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure.	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal line and one side on a vertical line, calculate the area of the figure.	Calculate the area of a triangle, rectangle, trapezoid, or composite of these three figures formed by linear equations and/ or determine the equations of the lines that bound the figure.	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations or equations of circles and/or determine the equations of the lines and circles that bound the figure.)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure.)	Calculate the area of a triangle, rectangle, trapezoid, circle, or composite of these figures formed by linear equations, linear inequalities, or conic equations and/or determine the equations of the lines and circles that bound the figure.
Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines.	Given 3 or 4 coordinate points that form a triangle or a rectangle with one side on a horizontal or vertical line, calculate the surface area and/or volume of the cone or cylinder formed by revolving the bounded region about either of the lines.	Given the equations of lines (at least one of which is horizontal or vertical) that bound a triangular, rectangular, or trapezoidal region, calculate the surface area and/or volume of the solid formed by revolving the region about the line that is horizontal or vertical.	Given the equations of lines or circles that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.	Given the equations of lines or circles or a set of inequalities that bound a triangular, rectangular, trapezoidal, or circular region, calculate the volume and/or surface area of the solid formed by revolving the region about a horizontal or vertical line.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

Using Linear Equations to Define Geometric Solids

ANSWERS

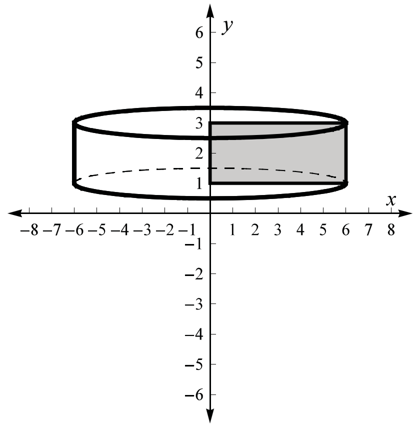
1. a.



b. $x = 0,\ x = 6,\ y = 1$ and $y = 3$

c. $P = 2(6) + 2(2) = 16$ units

$A = (6\text{units})(2\text{units}) = 12\ \text{units}^2$

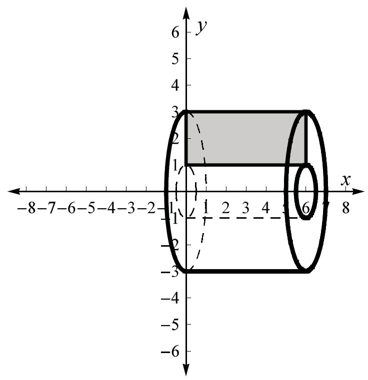


d. Cylinder with radius 6 units and height 2 units; the vertical cross-sections are rectangles; the horizontal cross-sections are circles.

e. $V = \pi r^2 h = \pi(6\,\text{units})^2(2\,\text{units}) = 72\pi\ \text{units}^3$

f. Large cylinder with a radius of 3 units and a height of 6 units which has a smaller cylinder with a radius of 1 unit and a height of 6 units removed from the center of the large cylinder.

$V = \pi(3\text{units})^2(6\text{units}) - \pi(1\text{units})^2(6\text{units}) = 48\pi\ \text{units}^3$



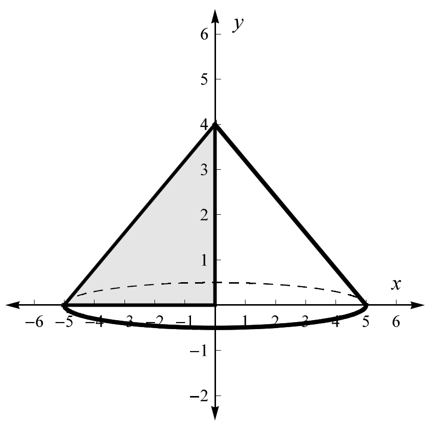
2. a. $y = \dfrac{4}{5}x + 4;\ y = 0;\ x = 0$

b. $P = 5\,\text{units} + 4\,\text{units} + \sqrt{41}\ \text{units} = 9 + \sqrt{41}\ \text{units}$

c. $A = \dfrac{1}{2}(5\,\text{units})(4\,\text{units}) = 10\,\text{units}^2$

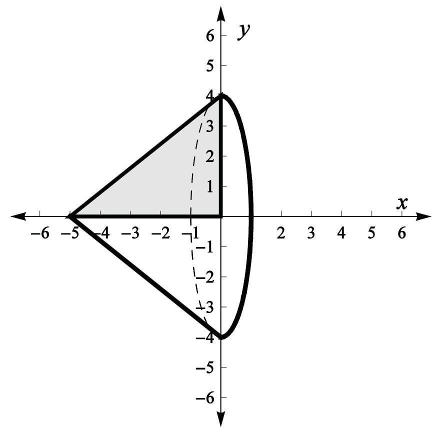
d. A cone with $r = 5\,\text{units}$ and $h = 4\,\text{units}$

Volume $V = \dfrac{1}{3}\pi(5\,\text{units})^2(4\,\text{units}) = \dfrac{100}{3}\pi\ \text{units}^3$

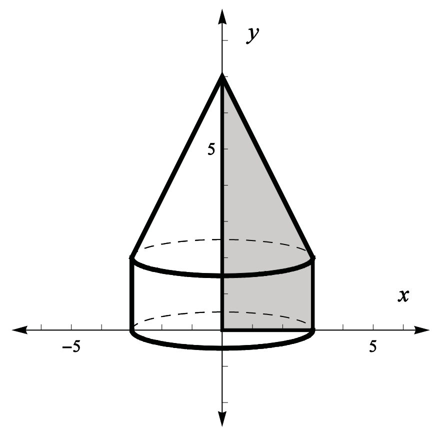


Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

e. The cone formed by revolving about the x-axis would have a smaller volume than the cone formed by revolving about the y-axis. The radius in the volume formula for a cone is squared so has a greater effect on the volume than the height. The radius of the cone in part (e) is 4 while the radius of the cone in part (d) is 5 which results in a smaller volume for the cone in part (e).



3. a. $f(x) = -\frac{5}{3}(x-9) - 8$; $g(x) = 0$; $x = 3$



b. A trapezoid with vertices (0, 7); (0, 0); (3, 0); (3, 2)

c. $P = \left(12 + \sqrt{34}\right)$ units

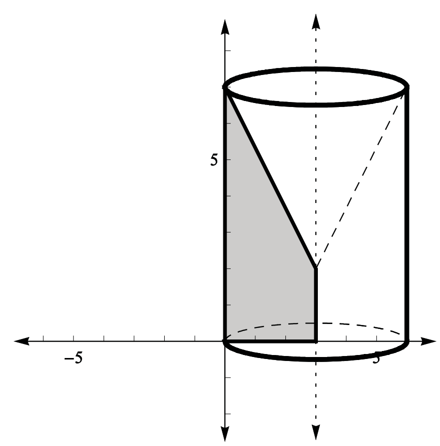
$$A = \frac{1}{2}(3\,\text{units})(7\,\text{units} + 2\,\text{units}) = \frac{27}{2}\,\text{units}^2$$

d. This solid is a cone with a radius of 3 units and a height of 5 units sitting on top of a cylinder with a radius of 3 units and a height of 2 units.

$$V = \pi r^2 h + \frac{1}{3}\pi r^2 h$$

$$V = \pi(3\,\text{units})^2(2\,\text{units}) + \frac{1}{3}\pi(3\,\text{units})^2(5\,\text{units}) = 33\pi\,\text{units}^3$$

e. This solid is a cylinder of height 7 units and radius 3 units with a cone of height 5 units and radius 3 units removed from the cylinder.

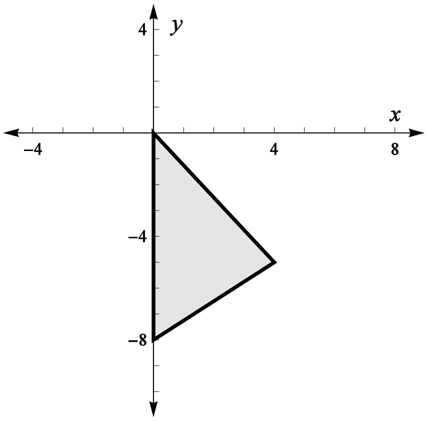


$$V = \pi r^2 h - \frac{1}{3}\pi r^2 h$$

$$V = \pi(3\,\text{units})^2(7\,\text{units}) - \frac{1}{3}\pi(3\,\text{units})^2(5\,\text{units}) = 48\pi\,\text{units}^3$$

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

4. a.

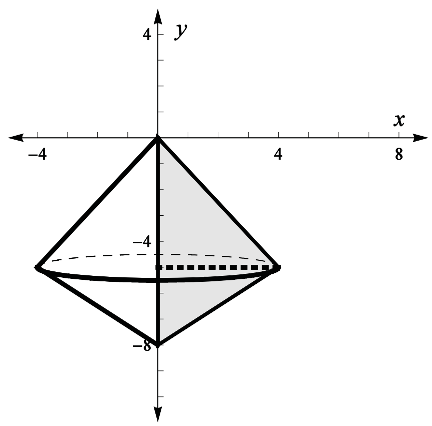


b. $\frac{-5}{4}x = \frac{3}{4}(x-4)-5$; so $x = 4$ and $y = -5$

The region forms a triangle whose vertices are $(0, 0)$; $(0, -8)$; $(4, -5)$ with a base of 8 units that lies on the y-axis and a height of 4 units.

$$A = \frac{1}{2}(8\,\text{units})(4\,\text{units}) = 16\,\text{units}^2$$

c. The solid consists of two stacked cones with a common base.
The cone with its vertex up has a radius of 4 units and a height of 5 units.
The cone with its vertex down has a radius of 4 units and a height of 3 units.



$$V = \frac{1}{3}\pi(4\,\text{units})^2(5\,\text{units}) + \frac{1}{3}\pi(4\,\text{units})^2(3\,\text{units})$$

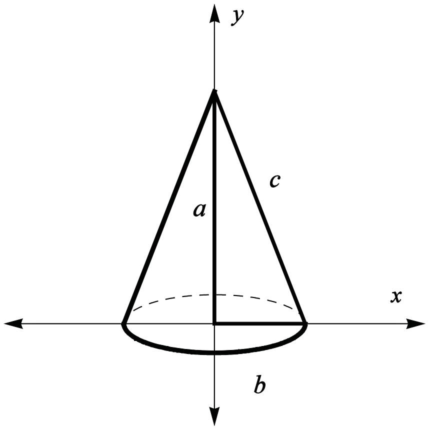
$$V = \frac{80}{3}\pi\,\text{units}^3 + 16\pi\,\text{units}^3$$

$$V = \frac{128}{3}\pi\,\text{cubic units}$$

TEACHER PAGES

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

5. a. $A=\frac{1}{2}ba$ units^2

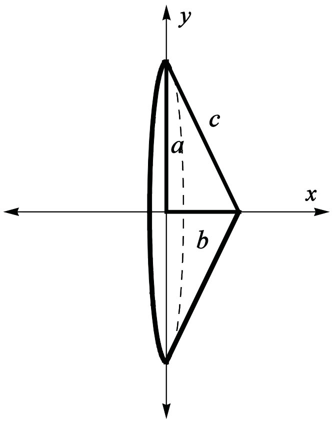


b. A cone with a radius *b* and a height *a*

$V=\frac{\pi}{3}b^2a$

c. A cone with a radius *a* and a height *b*

$V=\frac{\pi}{3}a^2b$



d. If the volume of the cone in part b is V_1 and the volume of the cone in part c is V_2, since you know that the volume of the cones are equal then $V_1=V_2$.

$V_1=V_2$

$\frac{1}{3}\pi(b)^2(a)=\frac{1}{3}\pi(a)^2b$

$b^2a=a^2b$ where $a\neq 0$ and $b\neq 0$

$a=b$

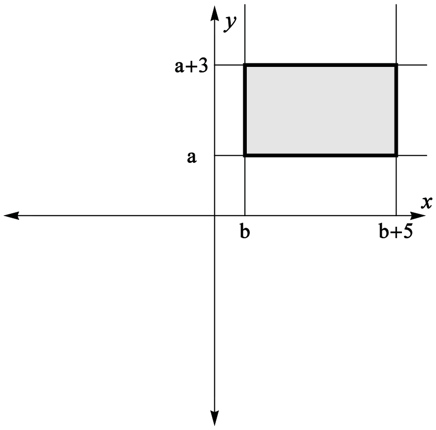
e. The solid is a cylinder of height *a* units and radius *b* units with a cone removed from the cylinder.

$V=\pi b^2a-\frac{1}{3}\pi b^2a$

$V=\frac{2}{3}\pi ab^2$ units^3

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

6. a.

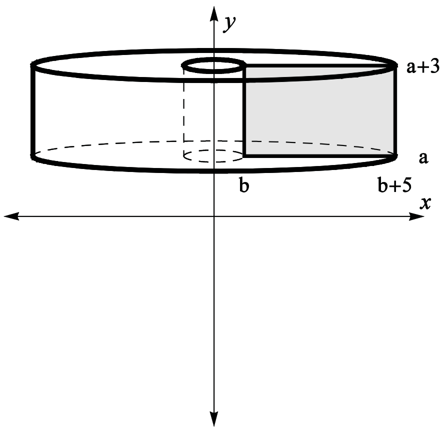


b. (b,a), $(b,a+3)$, $(b+5,a)$, $(b+5,a+3)$

c. Rectangle; $A = (5\,\text{units})(3\,\text{units})$ = 15 units^2

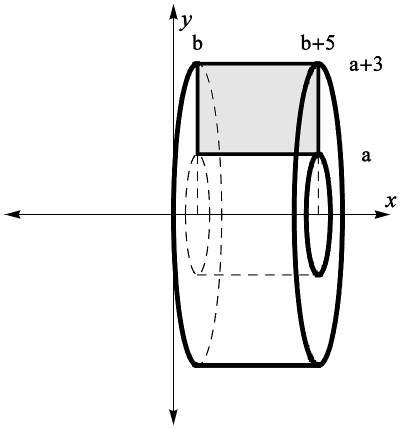
d. This solid consists of a large cylinder with a radius of $b+5$ units and a height of 3 units which has a smaller cylinder with a radius of b units and a height of 3 units removed from the center of the large cylinder.

$$V = \pi(b+5)^2(3) - \pi(b)^2(3)$$
$$V = 3\pi(b^2 + 10b + 25 - b^2)$$
$$V = (30\pi b + 75\pi)\ \text{units}^3$$



e. This solid consists of a large cylinder with a radius of $a+3$ units and a height of 5 units which has a smaller cylinder with a radius of a units and a height of 5 units removed from the center of the large cylinder.

$$V = \pi(a+3)^2(5) - \pi(a)^2(5)$$
$$V = 5\pi(a^2 + 6a + 9 - a^2)$$
$$V = (30\pi a + 45\pi)\ \text{units}^3$$



TEACHER PAGES

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

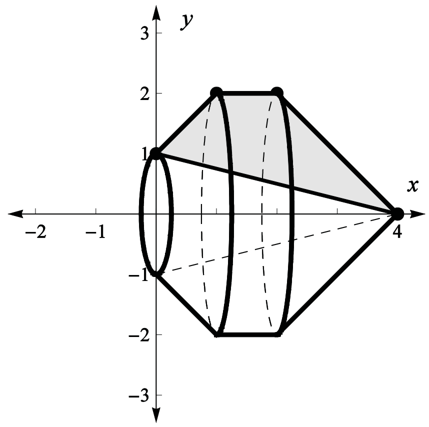
7. a. $P = 1\text{unit} + \sqrt{2}\text{ units} + 2\sqrt{2}\text{ units} + \sqrt{17}\text{ units} = \left(1 + 3\sqrt{2} + \sqrt{17}\right)\text{units}$

Begin by extending the line segment through $(1, 2)$ and $(0, 1)$ to the *x*-axis to form an isosceles trapezoid with bases of 5 and 1.

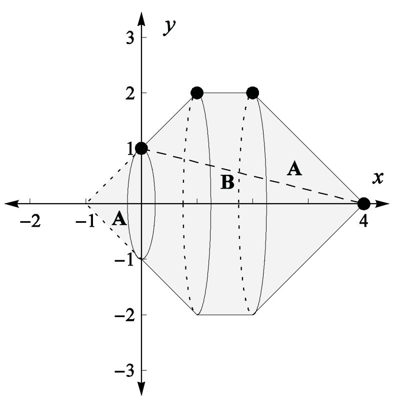
$$A = \frac{1}{2}(5\text{units} + 1\text{unit})(2\text{unit}) - \frac{1}{2}(5\text{ units})(1\text{ unit}) = 3.5\text{ units}^2$$

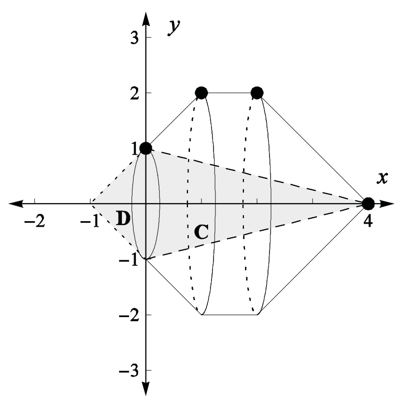
b. $y = x + 1$; $y = 2$; $y = 4 - x$; $y = -\frac{1}{4}x + 1$

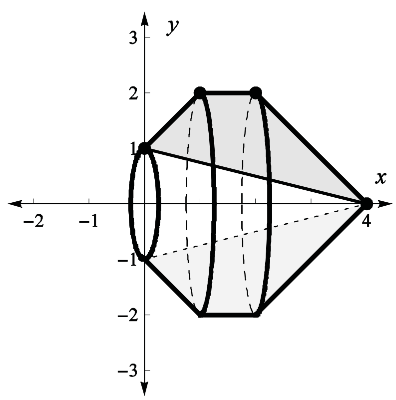
c.



d. To determine the volume of the figure when the region is revolved about the *x*-axis, first draw an auxiliary line $y = x + 1$ to extend the planar region to the point $(-1, 0)$. By drawing the auxiliary line, the figure created has two congruent triangles with a rectangle between the triangles. When the region is revolved about the *x*-axis, there are two cones, Cone A, of equal volume and a cylinder, Cylinder B, between the two cones. Determine the volume of both cones and the volume of the cylinder. Next, calculate the volume of the cone, Cone C, formed from the line $y = -\frac{1}{4}x + 1$ when revolved about the *x*-axis and then determine the volume of the cone, Cone D, formed by the auxiliary line $y = x + 1$ when revolved about the *x*-axis. To determine the volume when the planar region is revolved about the *x*-axis, use the sum of the two cones, Cone A, and cylinder, Cylinder B and subtract the volume of the two cones, Cones C and D which is defined by the equation $V_R = 2V_A + V_B - V_C - V_D$.

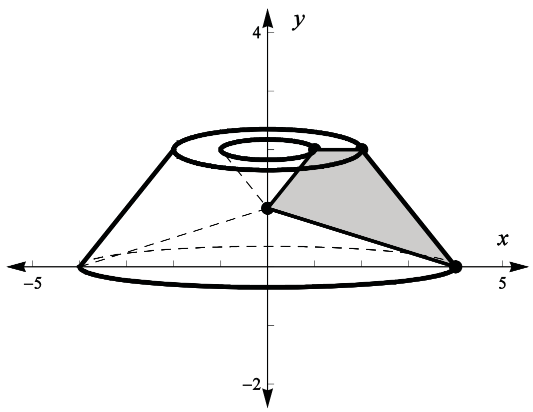




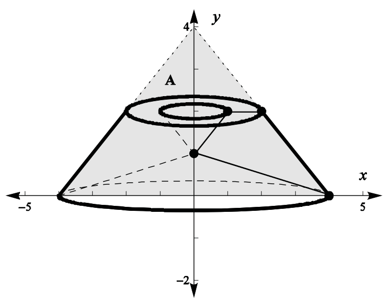


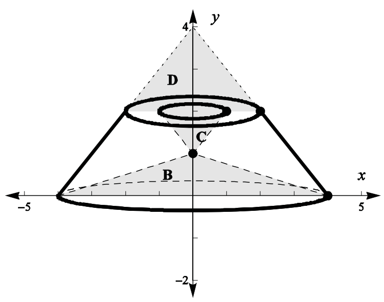
Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

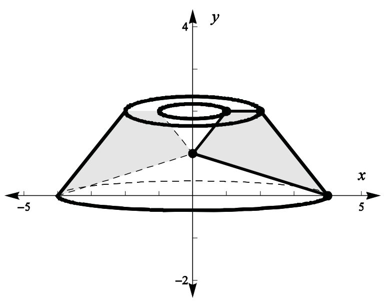
e.



f. To determine the volume of the figure when the region is revolved about the *y*-axis, first draw an auxiliary line $y = -x + 4$ to extend to the point $(0, 4)$ to visualize a cone formed when the planar region is revolved about the *y*-axis, Cone A. Determine the volume of the cone formed which has a radius of 4 and a height of 4. Determine the volume of the cone removed from the base, Cone B, which has a radius of 4 and a height of 1. Determine the volume of the inverted cone, Cone C, that has a radius of 1 and a height of 1. Determine the volume of the cone that was formed when drawing the auxiliary line $y = -x + 4$, Cone D, that has a radius of 2 and a height of 2. To compute the volume when the planar region is revolved about the *y*-axis, subtract the volume of Cones B, C and D from the volume of Cone A which is defined by the equation $V_R = V_A - V_B - V_C - V_D$.







g. Revolved about the *x*-axis

$$V_A = \frac{1}{3}\pi(2\,\text{units})^2(2\,\text{units}) = \frac{8}{3}\pi\,\text{units}^3$$

$$V_B = \pi(2\,\text{units})^2(1\,\text{unit}) = 4\pi\,\text{units}^3$$

$$V_C = \frac{1}{3}\pi(1\,\text{unit})^2(4\,\text{units}) = \frac{4}{3}\pi\,\text{units}^3$$

$$V_D = \frac{1}{3}\pi(1\,\text{unit})^2(1\,\text{unit}) = \frac{1}{3}\pi\,\text{units}^3$$

$$V_R = 2V_A + V_B - V_C - V_D$$

$$V_R = 2\left(\frac{8}{3}\pi\right) + 4\pi - \frac{4}{3}\pi - \frac{1}{3}\pi = \frac{23}{3}\pi\,\text{units}^3$$

Revolved about the *y*-axis

$$V_A = \frac{1}{3}\pi(4\,\text{units})^2(4\,\text{units}) = \frac{64}{3}\pi\,\text{units}^3$$

$$V_B = \frac{1}{3}\pi(4\,\text{units})^2(1\,\text{unit}) = \frac{16}{3}\pi\,\text{units}^3$$

$$V_C = \frac{1}{3}\pi(1\,\text{unit})^2(1\,\text{unit}) = \frac{1}{3}\pi\,\text{units}^3$$

$$V_D = \frac{1}{3}\pi(2\,\text{units})^2(2\,\text{units}) = \frac{8}{3}\pi\,\text{units}^3$$

$$V_R = V_A - V_B - V_C - V_D$$

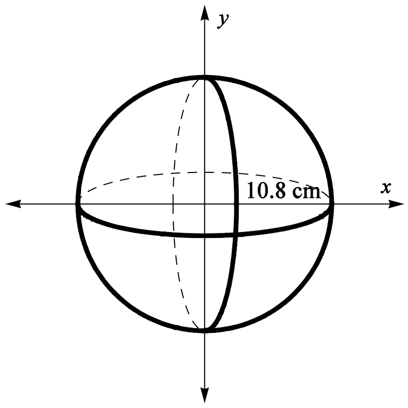
$$V_R = \frac{64}{3}\pi - \frac{16}{3}\pi - \frac{1}{3}\pi - \frac{8}{3}\pi = 13\pi\,\text{units}^3$$

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

8. a. sphere

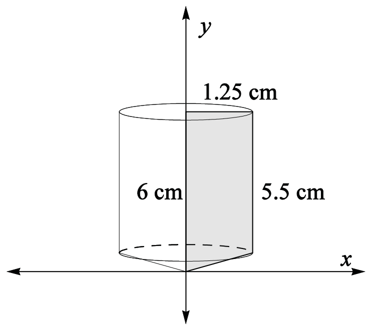
b. $C = 2\pi(10.8\,\text{cm}) = 67.858\,\text{cm}$

c. $V_s = \frac{4}{3}\pi(10.8\,\text{units})^3 = 5276.669\,\text{cm}^3$



d. The drill bit for thumb hole is a cylinder and a cone with a shared base whose radius is 1.25 cm. The height of the cylinder is 5.5 cm; the height of the cone is 0.5 cm.

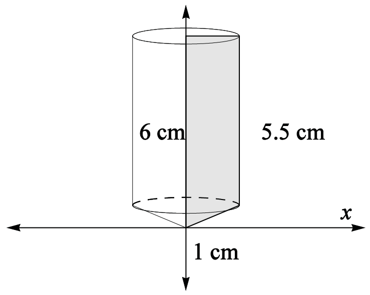
$$V_t = \pi(1.25\,\text{cm})^2(5.5\,\text{cm}) + \frac{1}{3}\pi(1.25\,\text{cm})^2(0.5\,\text{cm})$$
$$= 27.816\,\text{cm}^3$$



Volume removed for thumb hole

The drill bit for finger holes is a cylinder and a cone with a shared base whose radius is 1 cm. The height of the cylinder is 5.5 cm; the height of the cone is 05 cm.

$$V_f = \pi(1\text{cm})^2(5.5\,\text{cm}) + \frac{1}{3}\pi(1\text{cm})^2(0.5\,\text{cm})$$
$$= 17.802\ \text{cm}^3$$



Volume removed for finger holes

e. To calculate the total volume of the bowling ball after the holes are drilled, use the volume of the amount of resin used to fill the spherical mold to create the ball and subtract volume of the thumb hole resin removed; and then, since there are two finger holes drilled, subtract the volume of the finger holes twice. $V = V_s - V_t - 2V_f = 5213.248\,\text{cm}^3$

Note: Using intermediately rounded answers from parts (c) and (d) yields $V = 5213.249\ \text{cm}^3$

f. Because of the spherical shape, there are small polar caps removed as well as the volume of each drill bit. Our model did not account for these polar caps and thus we have a difference of 0.324 cm^3. Note: Using calculus the volume of the polar cap resulting from drilling the thumb 0.1783 cm^3 and the volume of the polar cap resulting from drilling each finger hole is 0.0729 cm^3.

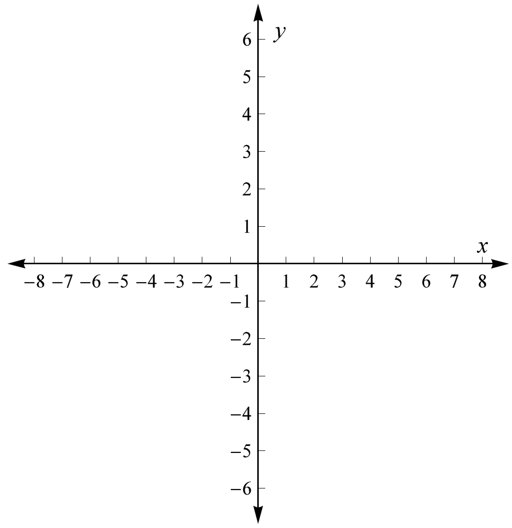
T E A C H E R   P A G E S

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

Using Linear Equations to Define Geometric Solids

1. a. Plot the points indicated in the table on the provided grid, and then draw horizontal and vertical line segments to connect the adjacent points.

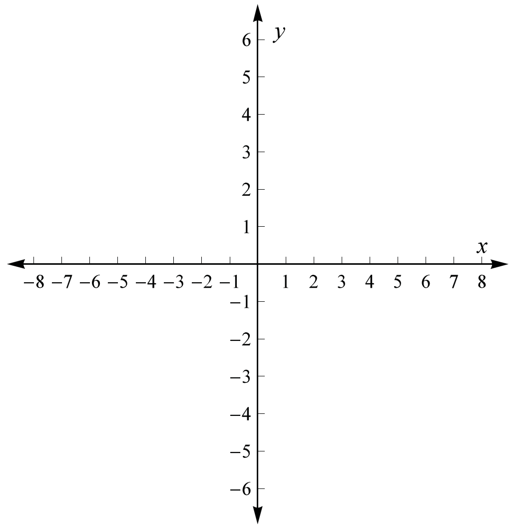
x	0	0	6	6
y	1	3	3	1



b. What are the equations for the four lines that bound this region?

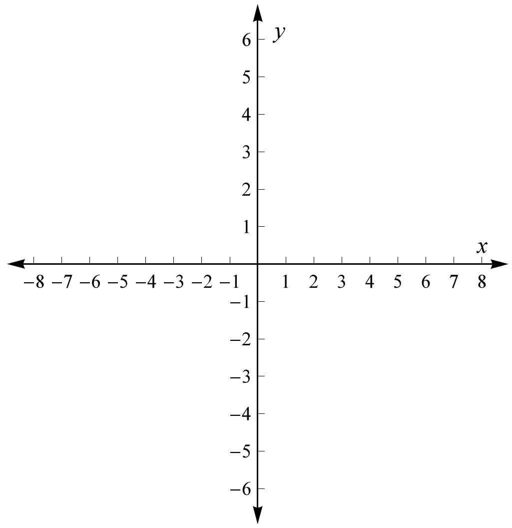
c. What is the perimeter and what is the area of the bounded region?

d. Draw and describe the geometric solid that is created by revolving the bounded region about the y-axis. What is the shape of the solid's vertical cross-sections? What is the shape of the solid's horizontal cross-sections?



e. What is the volume of the solid? State the answer in terms of π.

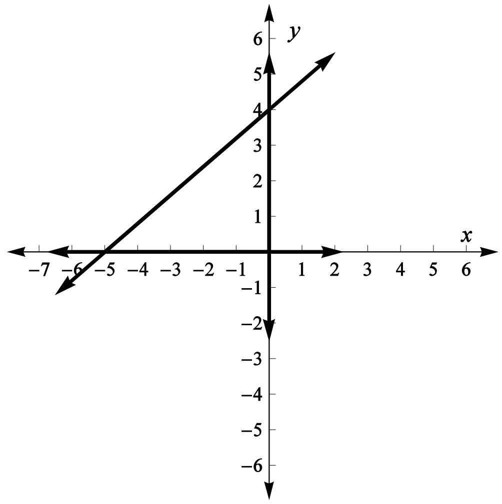
f. Draw and describe the geometric solid formed by revolving the bounded region about the x-axis. What is the volume of this solid? State the answer in terms of π.



Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

2. The region, R, is bounded by three lines.

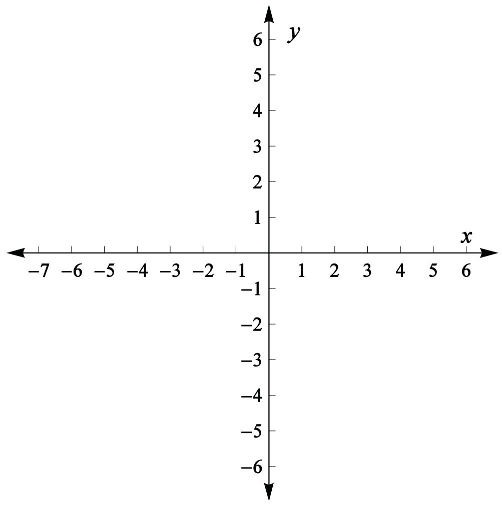
   a. What are the equations of the lines that define the bounded region?



   b. What is the perimeter of R?

   c. What is the area of R?

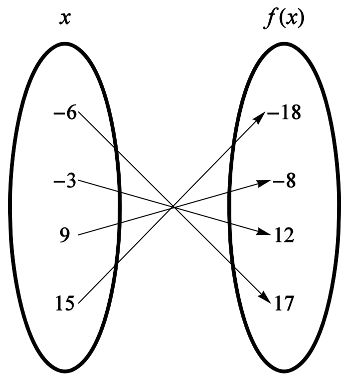
   d. Draw and describe the geometric solid formed by revolving region R about the y-axis. What is the volume of this solid? State the answer in terms of π .



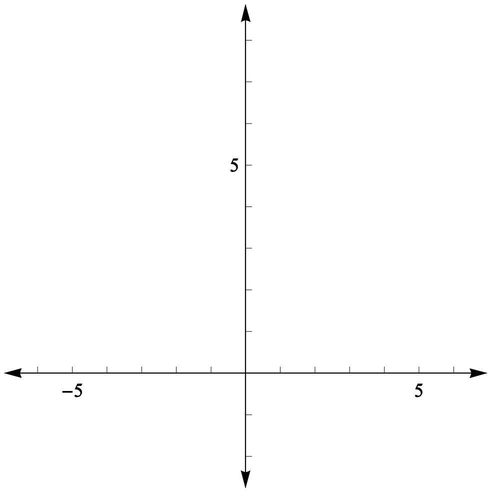
   e. Will the volume of the cone formed when the region R is revolved about the x-axis, instead of the y-axis, be greater, less, or the same? Explain the reasoning that leads to your answer.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

3. Given the following
   - a linear function $f(x)$
   - a linear function $g(x)$ for which $g(4)=0$ and $g(7)=0$
   - a vertical line that contains the point $(3, 7)$



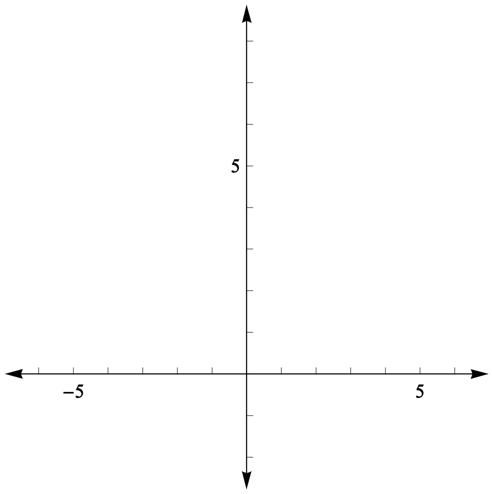
a. What are the equations of $f(x)$, $g(x)$, and the vertical line?



b. Graph and shade the region bounded by $f(x)$, $g(x)$, the vertical line, and the y-axis. Describe the region and list the coordinates of the vertices.

c. What is the perimeter and what is the area of the bounded region?

d. On the grid provided for part (b), sketch the solid that results from revolving the region about the y-axis. Describe the solid. What is the volume of this solid? State the answer in terms of π.

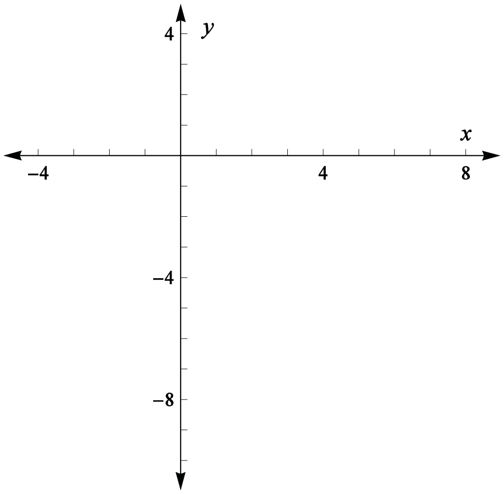


e. Draw and describe the geometric solid formed by revolving the bounded region about the vertical line $x=3$. What is the volume of this solid? State the answer in terms of π.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

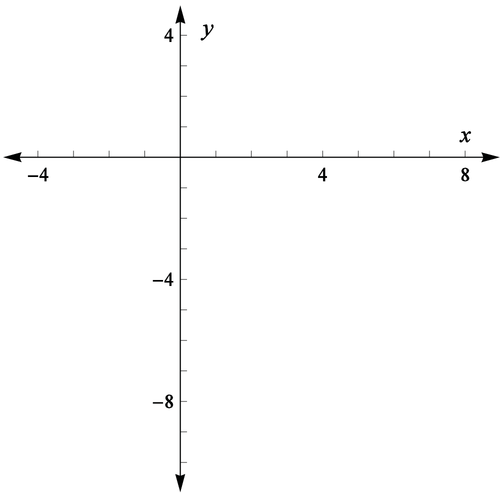
4. A region is bounded by the lines $y = -\frac{5}{4}x$, $y = \frac{3}{4}(x-4) - 5$, and $x = 0$.

   a. Graph each linear equation and shade the region bounded by these three lines.



   b. Describe the region, determine the vertices, and calculate the area of the region.

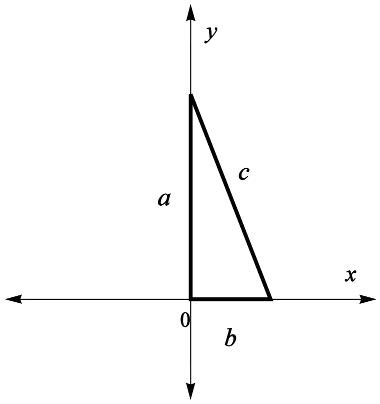
   c. Draw and describe the geometric solid that is created by revolving the bounded region about the *y*-axis. What is the volume of the solid? State the answer in terms of π.



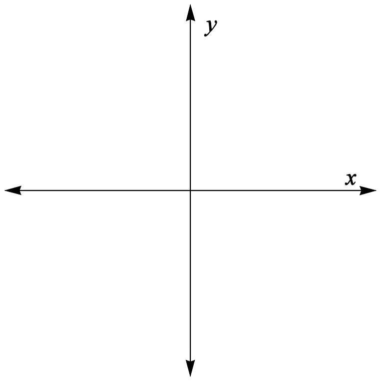
Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

5. A triangular region has sides of lengths a, b, and c, as shown on the graph.

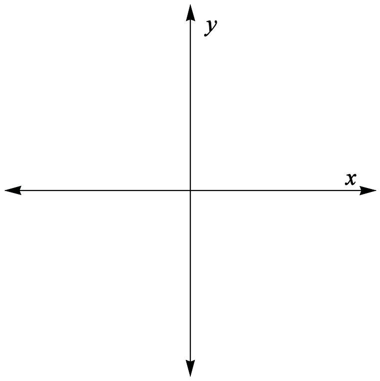
   a. What is the area of the region?



   b. Draw and describe the geometric solid that is created by revolving the region about the y-axis. What is the volume of this solid? State the answer in terms of π.

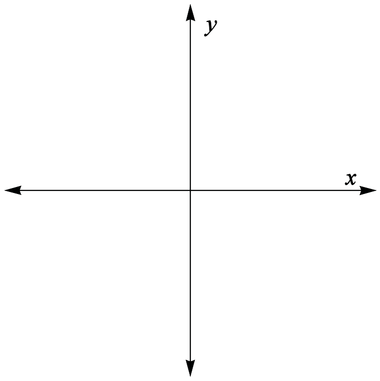


   c. Draw and describe the geometric solid that is created by revolving the region about the x-axis. What is the volume of this solid? State the answer in terms of π.



   d. If the volumes of the solids from part (b) and part (c) are the same, what is the relationship between a and b? Show the analysis that leads to your conclusion.

   e. If the region is revolved about the line $x = b$, describe how to determine the volume of the solid that is created. What is that volume?



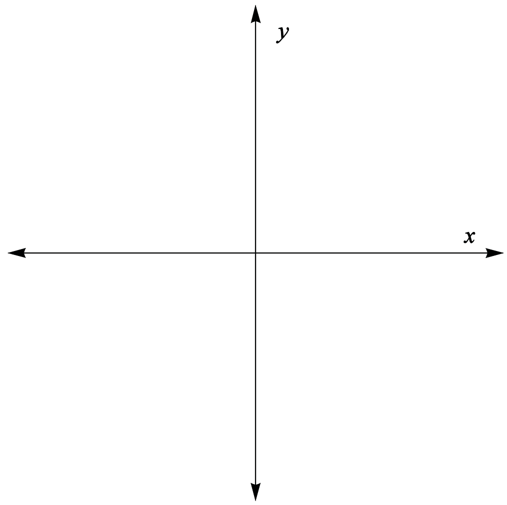
Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

6. A region is bounded by the lines
   $y = a,\ y = a + 3,\ x = b,\ x = b + 5$, where $a > 0$ and $b > 0$.

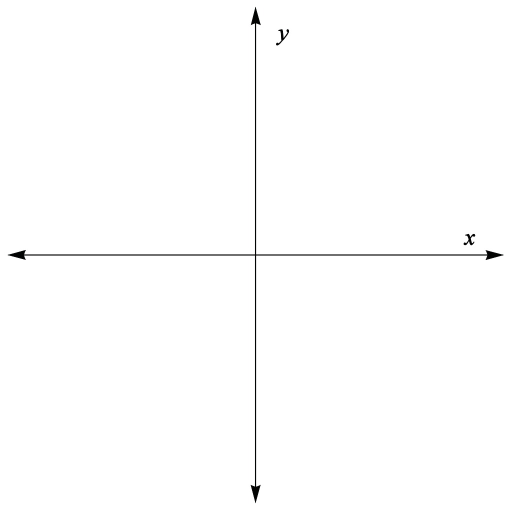
   a. Sketch a graph and shade the region bounded by these lines on the axes provided.

   b. List the vertices of the shape.

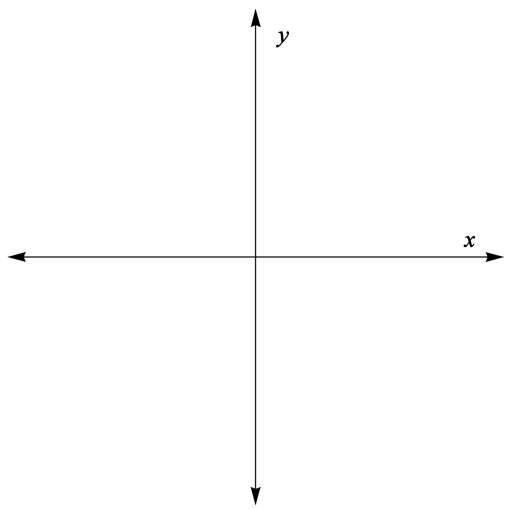
   c. Identify the figure formed by the shaded region and determine the area of the shaded region.



   d. Draw and describe the geometric solid that is created by revolving the region about the *y*-axis. Show that the volume of this solid is $(30\pi b + 75\pi)$ cubic units.

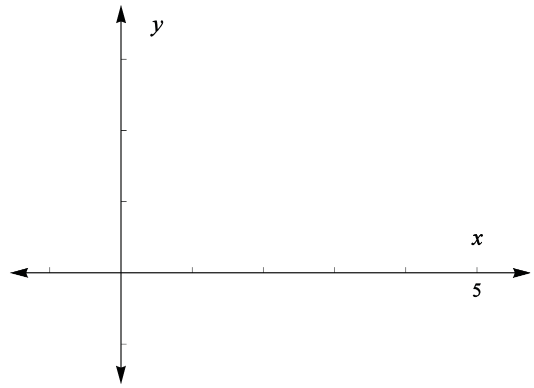


   e. Draw and describe the geometric solid formed by revolving the region about the *x*-axis. What is the volume of this solid? State the answer in terms of π.



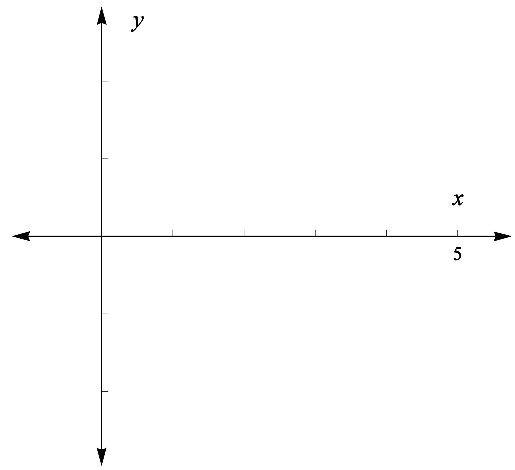
Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

7. Use the Mathematica demonstration to visualize the region with vertices $(0, 1)$, $(1, 2)$, $(2, 2)$ and $(4, 0)$. Create a sketch of the region.



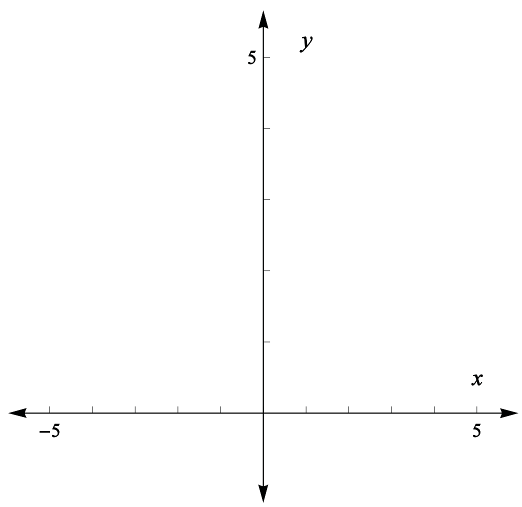
   a. What is the perimeter and what is the area of the bounded region?

   b. What is the system of equations that creates the bounded region?



   c. Use the Mathematica demonstration to revolve the figure about the *x*-axis. Draw a sketch of the figure created by the revolution on the grid.

   d. Explain a method that you could use to calculate the volume of the solid from part (c).



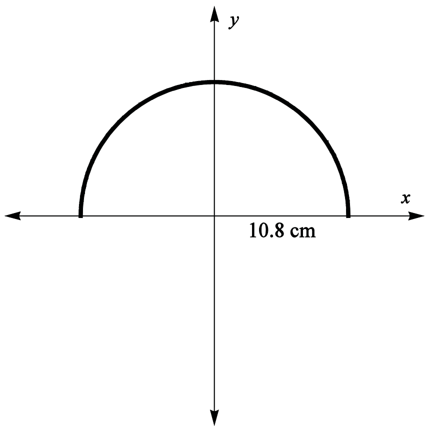
   e. Use the Mathematica demonstration to revolve the figure about the *y*-axis. Draw a sketch of the figure created by the revolution on the grid provided.

   f. Explain a method that you could use to calculate the volume of the solid from part (e).

   g. What is the volume of either the figure revolved about the *x*-axis or the figure revolved about the *y*-axis? State the answer in terms of π.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

8. Custom Bowling Balls, Inc. has designed a mold to make a new series of bowling balls out of resin. The mold has a radius of 10.8 cm as illustrated in the 2-dimensional drawing.

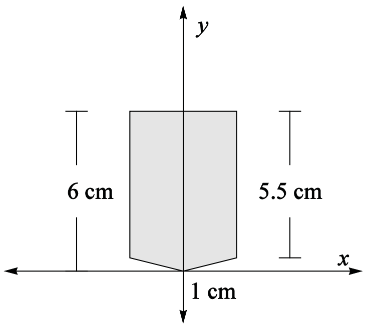


a. To view the three-dimensional mold, revolve the figure about the *x*-axis.
What three-dimensional figure does this revolution create?

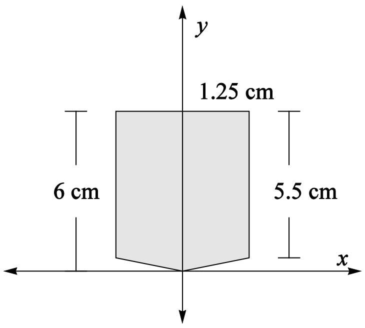
b. What is the circumference of the great circle of the bowling ball? Record the answer correct to three decimal places.

c. How much resin is needed to fill the mold? Record the answer correct to three decimal places.

After the bowling ball has been removed from the mold, Custom Bowling Balls Inc. will custom fit a customer's grip. After measuring the customer's fingers and thumb, two drill bits are selected for drilling the holes. The two-dimensional cross-sections for each drill bit used for one customer are shown.



Drill bit for finger holes

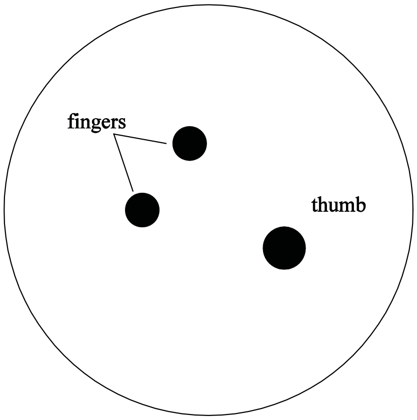


Drill bit for thumb hole

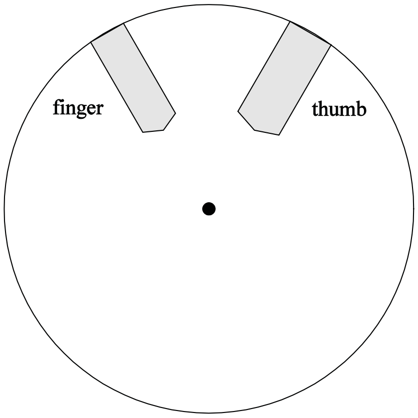
d. To model the hole created by drilling, revolve each of these figures about the *y*-axis. Describe the shapes of the holes that are created. Based on this model, what is the volume of the portion of the ball that will be removed by each bit?

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.

The company drills a hole for the thumb and two holes for the fingers as shown in the first figure and removes the resin. The customer's grip indicates that the finger and thumb holes should be drilled directly toward the center of the bowling ball, as shown in the second figure, which is referred to as "zero pitch."



Top–down view
showing the placement of
the finger and thumb holes



Cross–sectional view
showing the thumb and one finger

e. What is the approximate total volume of the bowling ball after the resin has been removed? Record the answer correct to three decimal places.

f. The actual volume of the bowling ball accurate to three decimal places is 5212.924 cm^3. What is the difference between your answer to part (e) and the true volume of the ball? Explain why this difference occurs.

Copyright © 2013 National Math + Science Initiative. This work is made available under a Creative Commons Attribution-NonCommercial-NoDerivs 3.0 United States license, http://creativecommons.org/licenses/by-nc-nd/3.0/us/deed.en_US.