LESSON 2
6th and 7th grade lesson

FINDING PI

ABOUT THIS LESSON

In this lesson, students engage in a handson activity which leads them to discover that, regardless of the size of a circle, the circumference of the circle divided by its diameter is equal to the value of pi (π) Students use ribbon to measure the circumference and diameter of circular objects. Precise measurement and reporting is required during this activity. After measuring diameters and circumferences of several circular objects, students record the diameter, circumference, and $C \div d$, in a table and then construct a graph using their measurements. Through analyzing the data in both the graphical and numerical forms, students observe the rate of change relationship when circumference is divided by diameter.

OBJECTIVES

Students will
- precisely measure circular objects.
- create scatterplots.
- determine the relationship of pi to the circumference and diameter of a circle.

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards

7.RP.2b: Recognize and represent proportional relationships between quantities.
(b) Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. *See questions 11, 19, 21*

LEVEL

Grade 6 or Grade 7 in a unit involving measurement and circumference

MODULE/CONNECTION TO AP*

Rate of Change: Average and Instantaneous

*Advanced Placement and AP are registered trademarks of the College Entrance Examination Board. The College Board was not involved in the production of this product.

MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

Reinforced/Applied Standards

7.RP.2a: Recognize and represent proportional elationships between quantities.
(a) Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin. *See questions 19-21*

7.RP.2d: Recognize and represent proportional relationships between quantities.
(d) Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where r is the unit rate. *See questions 10, 20*

7.G.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. *See questions 16-19*

6.SP.3: Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number. *See questions 11-12*

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.5: Use appropriate tools strategically. *Students use ribbon to measure the circumferences of various circular objects and to create a graph of circumference versus diameter.*

MP.6: Attend to precision. *Students determine the value of π and consider π as a precise unit.*

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:
*   Measure a diameter
*   Plot points in the first quadrant
*   Write and simplify ratios

ASSESSMENTS

The following types of formative assessments are embedded in this lesson:
*   Students summarize a process or procedure.

The following additional assessments are located on our website:
*   Rate of Change: Average and Instantaneous – 6th Grade Free Response Questions
*   Rate of Change: Average and Instantaneous – 6th Grade Multiple Choice Questions
*   Rate of Change: Average and Instantaneous – 7th Grade Free Response Questions
*   Rate of Change: Average and Instantaneous – 7th Grade Multiple Choice Questions

MATERIALS AND RESOURCES

*   Student Activity pages
*   Circular objects such as cans and lids
*   Four different colors of ribbon for each group
*   Rulers
*   Scissors
*   Butcher paper
*   Two copies of the tape measure page for each group to be used prior to the day of the lesson (Warning: Since copiers may change the dimensions of the tape measures on the page, measure a test copy. If the size is incorrect, adjust the scaling on the copier to correct the size.)
*   Cellophane tape
*   Glue
*   Scientific or graphing calculators

TEACHING SUGGESTIONS

Several days before teaching this lesson, ask each student to bring one circular object from home to be used in the lesson. On the day before the lesson, give each group two copies of the tape measure page from the end of the lesson. Have the students cut the page along the dotted lines to create 5 strips and then tape the strips together so that the repeated numbers line up with one copy on top of the other copy. These tapes will be used as the vertical and horizontal scales for the graph that the groups construct on butcher paper.

The length of butcher paper needs to be at least 15 centimeters more than the circumference of the largest circular object, and the width of the paper needs to be at least 15 centimeters more than the largest diameter. On the day of the lesson, students should draw a y-axis about 10 centimeters from the left side of the paper and an x-axis about 10 centimeters above the bottom of the paper. The paper tape measures are to be taped onto the graph as the vertical and the horizontal scales for the axes.

On the day the lesson is to be completed, provide each group with a pair of scissors and four pieces of ribbon that are long enough to wrap around the circumference of each object. Provide a ruler or a tape measure marked in centimeters for measuring the diameters and the length of the ribbons that have been wrapped around the circumference of the circular objects.

Demonstrate the procedures for measuring the diameter, for wrapping the ribbon around the object to measure the circumference, for cutting and measuring the ribbon, and for placing the ribbon on the butcher paper graph. Once the students understand the directions, have them work in groups to complete the activity.

A discussion to summarize the learning that takes place in the activity is necessary prior to having students write individual summaries. Students should understand that the first coordinate of each point represents the diameter of a circular object and that the second coordinate represents the circumference of the same object. Discuss that the ratio of the change in the circumference

to the change in the diameter of their circular objects should be the value of pi. Because of the limitations of measurements, students' ratios may not always be exactly the same, so calculating the exact decimal value of pi using this method is not possible. Students will enjoy looking at the value of pi as given on a calculator, so be prepared to explain that even a calculator does not show all of the digits of pi. Discuss the fact that pi is a non-terminating, non-repeating decimal – an irrational number. The purpose of this lesson, and the most important concept that students need to understand from this activity, is that the slope of the line is the change in the circumference divided by the change in the diameter. Making the connection between slope and rate of change will benefit students as they progress from middle school mathematics to algebra.

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7

contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

| 6TH GRADE SKILLS/ OBJECTIVES	7TH GRADE SKILLS/ OBJECTIVES	ALGEBRA 1 SKILLS/ OBJECTIVES	GEOMETRY SKILLS/ OBJECTIVES	ALGEBRA 2 SKILLS/ OBJECTIVES	PRE-CALCULUS SKILLS/ OBJECTIVES
From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.	From graphical or tabular data or from a stated situation presented in paragraph form, calculate or compare the average rates of change and interpret the meaning.

TEACHER PAGES

FINDING PI

Answers

1. – 9. Student activity posters

10. Points will vary; however, answers should include that the ordered pair is (diameter, circumference).

11. answers vary but should be close to 3.14

12. arithmetic mean

13. 3.141592654

14. 3.14

15. inaccuracy of measurement

16. $\pi = C \div d$

17. $C = \pi d$

18. $d = \pi \div C$

19.

DIAMETER	CIRCUMFERENCE
1	π
2	2π
3	3π
4	4π
5	5π

20. The points form a straight line that passes through the origin. The values at the origin would show zero diameter and circumference.

21. For each increase of 1 unit in the diameter, the circumference increases by π.

22. Sample student response provided on the following page.
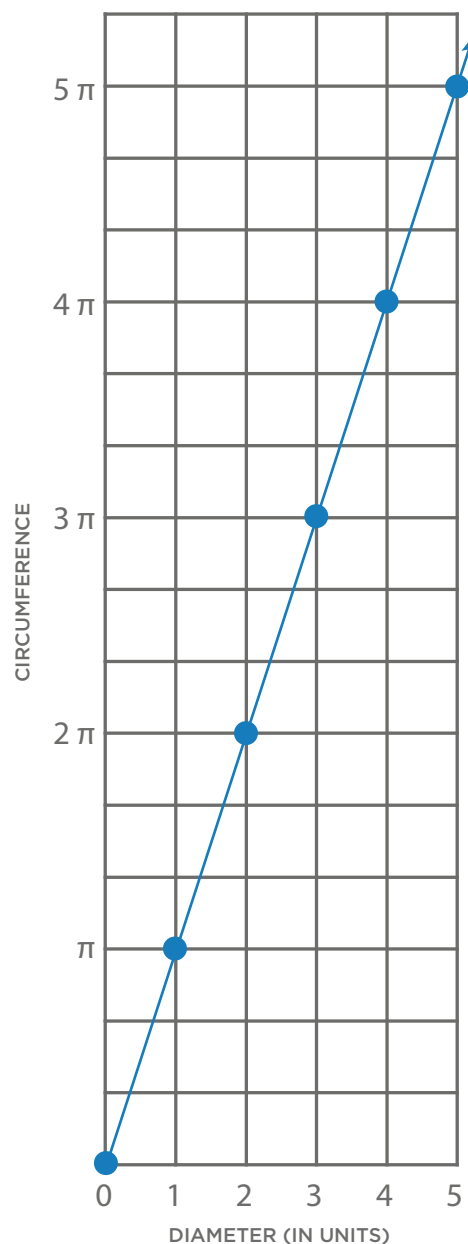
TEACHER PAGES

Summary of Finding Pi

Our class did an activity about how to find pi. We also learned how to find the diameter and the circumference of a circular object. You find the diameter by measuring across a circle through its center. You find the circumference by measuring the distance around the circle. The ribbons told how long the circumference was. The circumference of a circle is equal to π times its diameter.

If you know the circumference and don't know the diameter. You fin that by dividing circumference by π. If you know circumference and diameter to find π you divide circumference by diameter.

2021 NMSI
PI DAY MATH LESSONS
TEACHER KEY

NATIONAL
MATH + SCIENCE
INITIATIVE

LESSON 2
6th and 7th grade lesson

FINDING PI

Directions for creating the graph for this activity are provided in questions 1 – 9.

1. Using two copies of centimeter ruler pages, create two tape measures. Cut each page along the dotted lines and tape the strips together so that the 20 cm mark of the first strip lies exactly under the 20 cm mark of the second strip. Repeat the process of lining up the repeated units until a tape measure 100 cm long is created. The two strips will be used as the vertical and horizontal scales on your butcher paper graph.

2. On the butcher paper construct an x-axis and y-axis. Draw a vertical line for the y-axis 10 centimeters from the left side of the paper and a horizontal line for the x-axis 10 centimeters above the bottom of the paper. Line up one vertical tape just to the left of the vertical axis making sure that 0 on the tape is level with the bottom of the y-axis and then tape it in place. Put the second tape just below the horizontal axis. Line its zero up with the beginning of the x-axis and tape it in place. Ask your teacher to check your graph.

3. Compare the diameter of your circular objects by holding them against one another. Place your circular objects in order from the smallest diameter to the largest diameter. List the names of the objects in this order in the table above question 11.

4. Measure the diameter of the circular objects to the nearest millimeter and record the length of the diameter of each object in the table.

5. Wrap the ribbon around one of the circular objects and then cut the ribbon to the length of the circumference (the distance around the object).

6. Measure the length of the ribbon to the nearest millimeter. Now record this value in the circumference column of the table. Be sure to record it next to the diameter for the same object.

7. For the first object in the chart, locate the x-value on the graph that corresponds to its diameter. Glue the ribbon strip that you used to measure the circumference of that object vertically above the x-value of the diameter. Be sure that it is perpendicular to the x-axis and parallel to the y-axis.

8. Using the scale glued to the graph for the y-axis, check the length of each ribbon to see if it is close to the measurement recorded in the table for the circumference.

9. Repeat the process in steps 7 and 8 with the next object until you have graphed the diameter and circumference of all 4 of your circular objects.

10. Choose a point on the graph. Explain what this specific point means in terms of the circular object it represents. Write this answer on your poster.
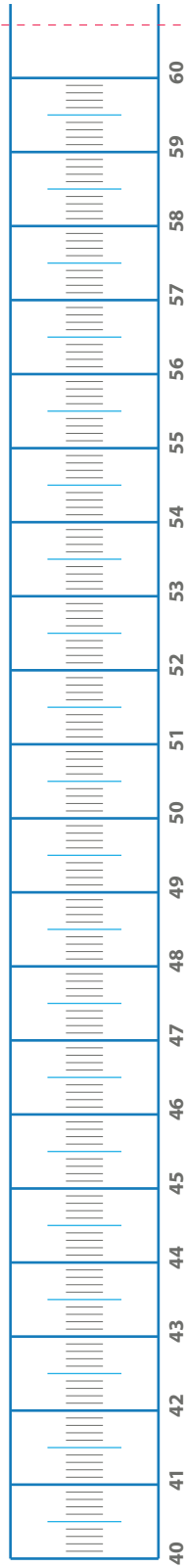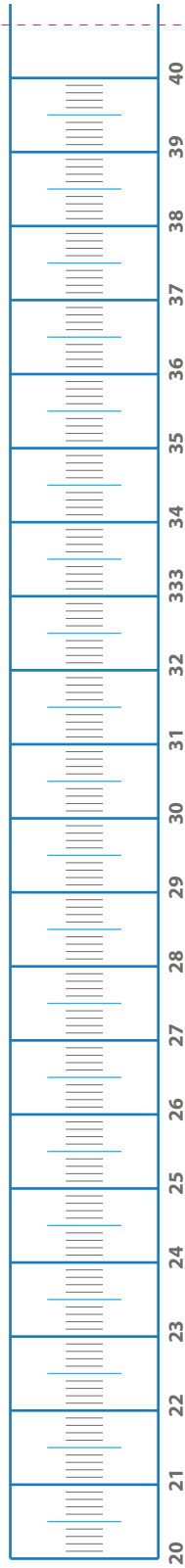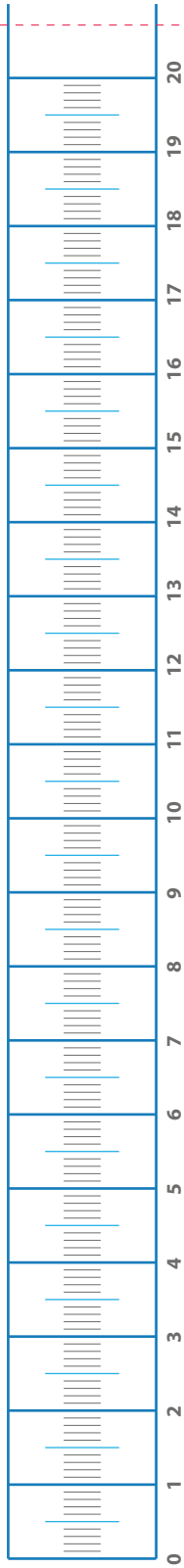
OBJECT NAME	CIRCUMFERENCE	DIAMETER	CIRCUMFERENCE ÷ DIAMETER

1. Complete the table by calculating the ratio of the circumference to the diameter, $C \div d$, of all four of your objects. Round your answers to the nearest hundredth. Average the four values in the $C \div d$ column and record this average in the space below.

2. What is the name for the measure of central tendency that you just calculated in question 11?

3. The value of π is a constant because its value is always the same. Locate the π button on your calculator and press enter. Record all the numbers that are displayed on your calculator.

4. What is the number, rounded to two decimal places, that is often used as the value of π ?

5. Is there a difference in the value you determined in question 11 and the rounded value we often use for π ? If there is, why do you think this difference exists?

6. Write an equation that can be used to determine the value of the constant, π , if you know the circumference and the diameter of a circular object.

7. Write an equation that can be used to determine the circumference of a circle if you know the value of the constant, π , and the diameter of a circular object.

8. Solve the equation in question 17 for d. In other words, write an equation that can be used to determine the diameter of a circular object if you know the circumference of the object and the value of π .

9. Complete the chart with exact answers (answers in terms of π) for the circumference. Plot the coordinates of your five points. Make sure that the diameter is the first coordinate (*x-axis*) and the circumference is the second coordinate (*y-axis*). The coordinates of each point will be (diameter, circumference).

DIAMETER	CIRCUMFERENCE
1	
2	
3	
4	
5	

10. Connect the points on the graph. Do your points connect in a straight line? If you were to extend your line, would the line pass through the origin (0,0) on your graph? What would the values at the origin represent?

11. For each increase of one unit in your diameter, what happens to the value of the circumference?

12. Write a paragraph describing the activity and your conclusions.

TEACHER PAGES

| 20 | 19 | 18 | 17 | 16 | 15 | 14 | 13 | 12 | 11 | 10 | 9 | 8 | 7 | 6 | 5 | 4 | 3 | 2 | 1 | 0 |

| 40 | 39 | 38 | 37 | 36 | 35 | 34 | 333 | 32 | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 |

| 60 | 59 | 58 | 57 | 56 | 55 | 54 | 53 | 52 | 51 | 50 | 49 | 48 | 47 | 46 | 45 | 44 | 43 | 42 | 41 | 40 |

| 80 | 79 | 78 | 77 | 76 | 75 | 74 | 73 | 72 | 71 | 70 | 69 | 68 | 67 | 66 | 65 | 64 | 63 | 62 | 61 | 60 |

TEACHER PAGES