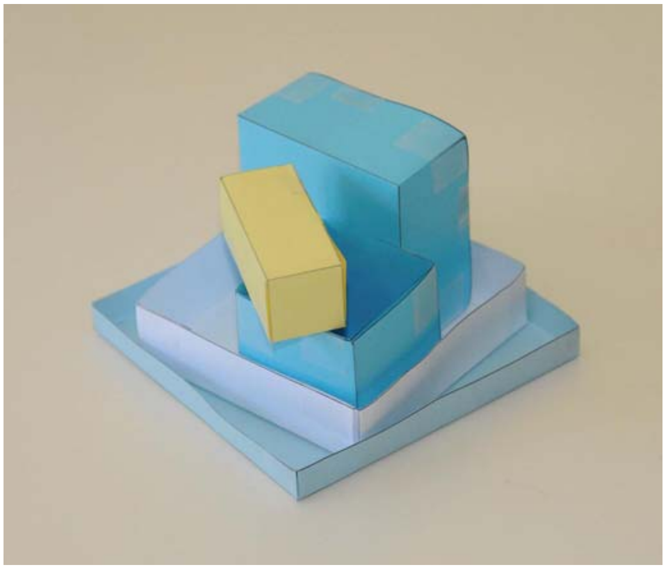

Box It Up

ABOUT THIS LESSON

This lesson provides a "hands on" approach to constructing an open-topped box with maximum volume. Using both square and rectangular sheets of paper, students cut squares from the corners of the paper and fold up the sides to create the boxes. Recording the dimensions and volumes of the resulting boxes in tables, on graphs, and as equations helps students analyze the volumes and generalize their relationships.

LEVEL

Grade 6 in a unit on volume

MODULE/CONNECTION TO AP*

Optimization: Area and Volume Applications

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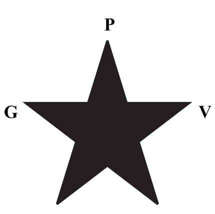
MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

OBJECTIVES

Students will

- create open-topped boxes from square and rectangular sheets of paper.
- calculate the volumes of open-topped boxes constructed by cutting squares from the corners of square and rectangular sheets of paper.
- determine the relationship between the height of the box and the length of the side of the square paper when the maximum volume occurs.
- write the equation for the volume of a box in terms of its height when congruent corners are removed from a rectangular sheet of paper.

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards

6.G.2: Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = lwh$ and $V = bh$ to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
See questions 1, 2d, 3a, 3j

Reinforced/Applied Standards

6.EE.2c: Write, read, and evaluate expressions in which letters stand for numbers.
(c) Evaluate expressions at specific values for their variables. Include expressions that arise from formulas in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). *For example, use the formulas $V = s^3$ and $A = 6\,s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.*
See questions 3g-i

5.G.2: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.
See questions 3b, 3j

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.8: Look for and express regularity in repeated reasoning.
Based on their numerical calculations in question 1, students recognize the ratio of the height of the box to the length of the original paper must be 1:6 in order to create a box of maximum volume from a square piece of paper.

Students who realize that the lengths and widths can be calculated directly from the dimensions of the paper and the height can eliminate building the box.

Students work with numerical examples to develop rules for the length and width of the box in terms of the height and an equation for volume in terms of the height.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Calculate volumes of rectangular prisms
- Graph points in the first quadrant

ASSESSMENTS

The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on our website:

- Optimization: Area and Volume Applications – 6^{th} Grade Free Response Questions
- Optimization: Area and Volume Applications – 6^{th} Grade Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
- Tape
- Scissors
- 5 copies of each of the squares with sides of 4 inches, 5 inches, and 6 inches provided at the end of the lesson
- Rulers
- 4 sheets of 8.5 inch by 11 inch paper
- Popcorn (optional)
- Graphing calculators (optional)
- Interactive applet that allows students to cut various size squares from the sides of different sized grid paper, form a rectangular prism, and calculate surface area and volume: http://mste.illinois.edu/users/carvell/3dbox/default.html#simulation

TEACHING SUGGESTIONS

Consider completing question 1 as a group activity, assigning each student a particular box. It is important that all students be given the opportunity to cut, construct, and measure boxes. Provide a sufficient number of sheets of paper so that all of the boxes can be constructed. Have some extras on hand in case there are mistakes made in cutting out the corners.

Discuss the procedure with the students before they begin so that they will understand how to construct each box. Hold up one of the sheets of paper that is 6 inches by 6 inches and demonstrate how to form a box by carefully cutting $\frac{1}{2}$ inch congruent squares from each corner. If the cut is made in only one direction, a tab can be formed, making the box easier to tape together. Fold up the sides and tape them to make a box without a top. Explain that the paper rulers provided on the pages at the end of the activity are scaled in $\frac{1}{12}$ inch units. They are helpful when measuring the required heights on the 5-inch square and the 4-inch square, since most rulers are not scaled to accurately measure $\frac{2}{3}$ inch or $\frac{5}{6}$ inch. Now ask the students how to determine the actual volumes of the boxes. Ask the students how they know these measurements are accurate based on the size of the original piece of paper. For the first box, the length should be $6-2\left(\frac{1}{2}\right)=5$ inches, the width should be $6-2\left(\frac{1}{2}\right)=5$, and the height should be $\frac{1}{2}$ inch. Record the dimensions in the chart and then have the students determine the volume.

Emphasize to students that for question 1, the actual maximum volume can be obtained using one of the lengths shown in the chart. Be sure students understand that the generalization in question 2 only applies when a square piece of paper is used.

After all groups have completed questions 1 – 2 and the generalizations have been discussed, introduce question 3. Tell the students that they will be able to fill their boxes with popcorn, so they want to create a box with the largest volume. After students have built the boxes for question 3, number the boxes on the bottom with the height of the box and then ask the students to guess which box will have the greatest volume. Record their guesses and post them. Let the students test their guesses by filling the boxes with popcorn and arranging the boxes in order by volume. When the students have ordered the boxes, compare the results from the groups and discuss the differences from the original guesses. When the students have completed the activity, compare the calculations and ask if the volume calculations confirm the same order as the popcorn. As an extension, students can enter the equation from question 3i into the graphing calculator and determine the dimension of the square corner which will produce an open-topped box with the maximum volume.

You may wish to support this activity with TI-Nspire™ technology. See *Working with Fractions and Decimals* and *Finding Points of Interest* in the NMSI TI-Nspire Skill Builders.

Suggested modifications for additional scaffolding include the following:

- 1 Provide the squares with the corners removed for each activity.
- 1b Provide the widths for the table.
- 1c Provide the first and last lengths and the third and fourth widths.
- 3a Provide the second and third widths for the table.

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Use formulas for perimeter, circumference, area, and/or volume to generate data.	Use formulas for perimeter, circumference, area, and/or volume to generate data.	Use formulas for perimeter, circumference, area, and/or volume to create algebraic models.	Use formulas for perimeter, circumference, area, and/or volume to create algebraic models.	Use formulas for perimeter, circumference, area, and/or volume to create algebraic models.	Use formulas for perimeter, circumference, area, and/or volume to create algebraic models.
Using graphical or numerical data, determine an optimal solution.	Using graphical or numerical data, determine an optimal solution.	Using an algebraic model, determine an optimal solution.	Using an algebraic model, determine an optimal solution.	Using an algebraic model, determine an optimal solution.	Using an algebraic model, determine an optimal solution.

Box It Up

1. Use square pieces of paper as described to form open-topped boxes by carefully cutting congruent squares from each corner. Use the measurements given for the heights of the boxes as the lengths of the sides of the squares to be cut from the corners. Making one cut at each corner to form a tab will make assembling the box easier. Complete the tables.

   a. Use a square piece of paper that is 6 inches by 6 inches to form each box.

Length of the box	**Width of the box**	**Height of the box**	**Volume of the box**
		$\frac{1}{2}$ inch	
		1 inch	
		$1\frac{1}{2}$ inches	
		2 inches	
		$2\frac{1}{2}$ inches	

   b. Use a square piece of paper that is 5 inches by 5 inches to form each box.

Length of the box	**Width of the box**	**Height of the box**	**Volume of the box**
		$\frac{2}{3}$ inch	
		$\frac{5}{6}$ inch	
		1 inch	
		$1\frac{1}{6}$ inches	
		$1\frac{1}{3}$ inches	

c. Use a square piece of paper that is 4 inches by 4 inches to form each box.

Length of the box	Width of the box	Height of the box	Volume of the box
		$\frac{1}{3}$ inch	
		$\frac{1}{2}$ inch	
		$\frac{2}{3}$ inch	
		$\frac{5}{6}$ inch	
		1 inch	

2. A pattern develops when open-topped boxes are made from square pieces of paper. Answer the following questions to determine this relationship.

a. Complete the chart, using the box with largest volume from each part of question 1.

Length of the Side of the Original Paper	Height of the Box with Maximum Volume
6 inches	
5 inches	
4 inches	

b. For the open topped box with maximum volume made from a square piece of paper, what is the ratio of the height of the box with maximum volume to the length of the side of the original paper? For any square sheet of paper, what is the ratio of the length of the square cutout to the length of the original square?

c. A larger square sheet of paper is to be used to make a box with the maximum volume. What is a possible length for the side of the sheet of paper, if the height of the box must be a whole number?

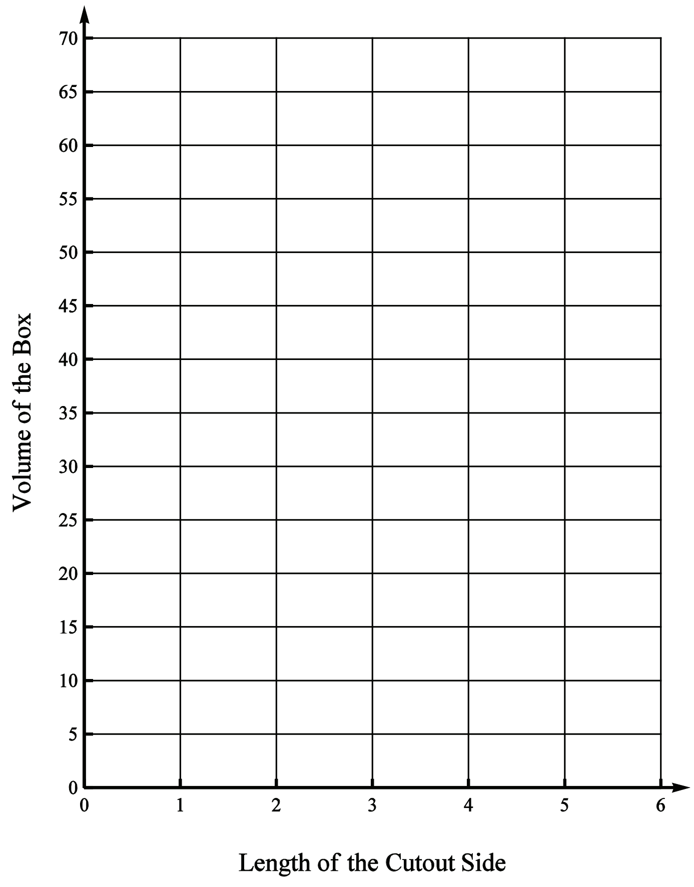
d. If you were given a square piece of metal sheeting with sides measuring 3 feet, what would be the size of the cutout corner that would give you the box with maximum volume? Determine the dimensions and maximum volume of this box made out of metal sheeting.

3. Use an 8.5 inch by 11 inch piece of paper to form an open-topped box by carefully cutting congruent squares from each corner. Use only whole number values for the lengths of these squares. The length of the box will be formed from the 11 inch side and the width of the box will be formed from the 8.5 inch side.

   a. Fold up the sides and tape them to make a box without a top. Measure the length, width, and height of the resulting box and complete the table.

Length of the box	**Width of the box**	**Height of the box**	**Volume of the box**

   b. Graph this data using the length of the cutout side as the independent variable (the *x*-axis) and the volume of the box as the dependent variable (the *y*-axis).



c. The length of the side of the cutout square becomes which dimension in the resulting box?

d. What is the largest square with whole number dimensions that can be cut out? Why is it not possible to cut out a square with a larger value?

e. Of the boxes in your table, what are the dimensions of the box that has the least volume? What is the volume of this box?

f. Of the boxes in your table, what are the dimensions of the box that has the greatest volume? What is the volume of this box?

g. Using the length of the original paper, 11 inches, and the height of the box, h, either explain how to determine the length of the box or write a rule for determining the length of the box.

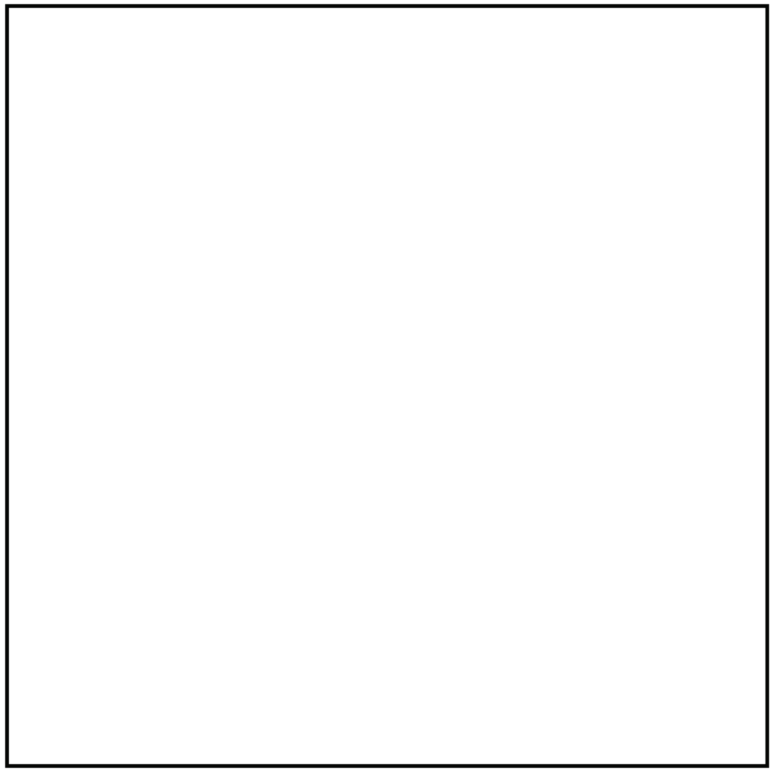
h. Using the width of the original paper, 8.5 inches, and the height of the box, h, either explain how to determine the width of the box, or write a rule for determining the width of the box.

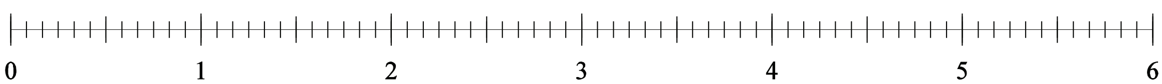
i. Write an equation for the volume of the box in terms of the height of the box.

j. Only whole numbers were used for the square cutouts of the boxes in the table. There are other boxes with volumes larger than any of these. Determine the dimensions and the volume of a box that will be greater than the ones that have been built. Plot this data point on your graph in part (b). Show your work.

Box It Up

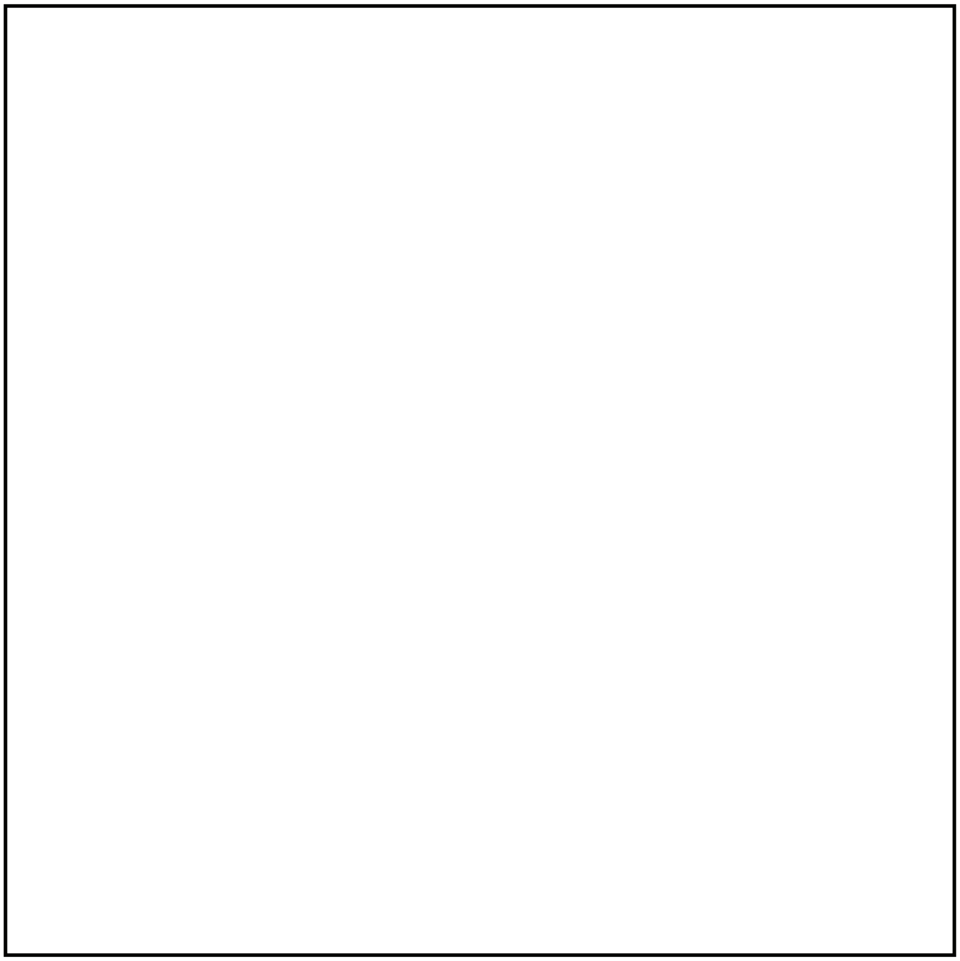
4-inch Square

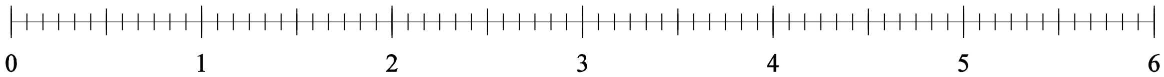




Box It Up

5-inch Square





Box It Up

6-inch Square