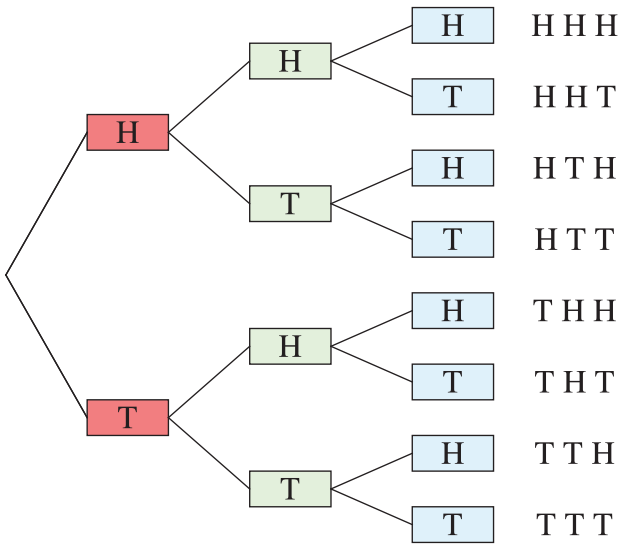

LEVEL

Grade 7 in a unit on probability

MODULE/CONNECTION TO AP*

Probability

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MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.

P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

Family Fun (Binomial Probability)

ABOUT THIS LESSON

This lesson introduces the concept of binomial probability using sample spaces and Pascal's triangle. The first situation in the activity contains a sample space small enough to be created with a tree diagram or a table, allowing the probabilities to be determined by counting. In later questions, the lesson uses the patterns in Pascal's triangle to determine probabilities for situations with large sample spaces. While focusing on probability, this lesson can be used to reinforce the idea that much of mathematics can be looked at as a study in patterns.

OBJECTIVES

Students will

- construct sample spaces to determine probability of simple and compound events.
- use sample spaces to determine conditional probabilities.
- refer to Pascal's triangle to determine binomial probabilities.
- use the patterns in Pascal's triangle to extend the number of rows.

TEACHER PAGES

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards

7.SP.7a: Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.
(a) Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.
For example, if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.
See questions 1-6, 11-14

7.SP.8a: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
(a) Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
See questions 1-6, 11-14

Reinforced/Applied Standards

7.SP.8b: Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
(b) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.
See chart and questions 7-10

7.SP.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
See questions 2-6, 11-13

6.SP.5a: Summarize numerical data sets in relation to their context, such as by:
(a) Reporting the number of observations.
See chart and questions 1, 8-13

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.7: Look for and make use of structure.
Students use the patterns existing in Pascal's triangle to extend its rows in order to determine probability for situations with large sample spaces.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Calculate simple probabilities
- Construct tree diagrams

ASSESSMENTS

The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on our website:

- Probability – 7th Grade Free Response Questions
- Probability – 7th Grade Multiple Choice Questions

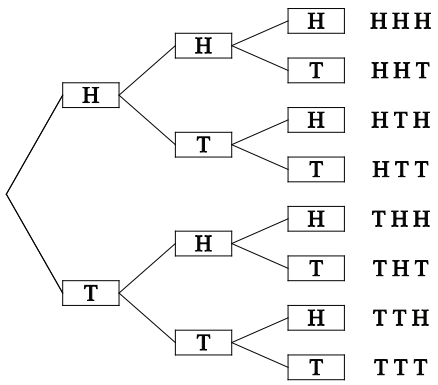
MATERIALS AND RESOURCES

- Student Activity pages

TEACHING SUGGESTIONS

Binomial probability requires that: (1) an event has exactly two outcomes, "bi", (2) the number of trials is fixed, "nom", (3) the trials are independent of one another, which means that the outcome of one event does not affect the outcome of the next event, "i", (4) the probabilities remain constant throughout the situation, "al". In this lesson, the situation with the family of four children represents an example of binomial probability. There are exactly two outcomes, boy or girl, the number of trials is fixed, four children, the trials are independent of one another, and the probabilities never vary no matter which family position is being considered. Even though the probability of a boy or a girl is not given, we will assume that both outcomes are equally likely throughout the situation.

Before beginning the activity, the teacher should guide students in listing the possible outcomes from tossing 3 coins. Some students understand this activity more clearly if 3 different coins are used, such as a penny, nickel, and dime. A tree diagram may also be helpful.



Sample Space

HHH HHT HTH HTT THH THT TTH TTT

Emphasize that TTH is a different outcome from HTT, etc. If three different types of coins have been used, this statement will make more sense to the students.

Question 6 involves conditional probabilities. Show students that the given condition, number of boys, limits the sample space of possible outcomes to a specific column.

For questions 7 – 13, Pascal's triangle is used as a tool to help students complete questions about sample spaces that would be time consuming to create. The "triangle" represents the number of ways an arrangement can be created. The numbers in the triangle are based on combinations and can be created algebraically. For example, in question 10, students use row 7 to determine the number of boy/girl outcomes for a family with 7 children. The number "21" represents the number of ways 3 boys and 4 girls can be arranged or the number of ways 3 girls and 4 boys can be arranged.

Students may need to discuss the patterns found in Pascal's triangle so that they can extend the triangle with additional rows. Students should notice that the sides of the triangle are ones and that each number inside the triangle is the sum of the two numbers diagonally above it.

Suggested modifications for additional scaffolding include the following:

Chart Provide the column for 3 boys and the last row of the table giving the total possible outcomes for each column. Check the table before the student progresses through the lesson.

11 Correlate the first 4 positions of the 7th row of Pascal's triangle, 1, 7, 21, 35 to the number of boys in a family of 7 children: 1 – no boys, 7 – exactly one boy, 21 – exactly two boys, 35 – exactly three boys. Use this row to assist the student in determining the probability of exactly three boys in a family with 7 children.

13 Provide rows 9 and 10 of Pascal's triangle.

14 Modify to ask the student to write one probability question or provide an example such as "probability that on a five question true/false quiz you randomly guess more than one question correctly" and have the student explain how to use Pascal's triangle to determine the probability.

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.
Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements using the general addition and multiplication rules.	Compute the probability of compound events and their complements using the general addition and multiplication rules.	Compute the probability of compound events and their complements using the general addition and multiplication rules.
Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.	Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.	Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.	Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.	Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.	Create and use sample spaces based on Venn diagrams, two-way tables, tree diagrams, and Pascal's triangle.

Family Fun (Binomial Probability)

Complete the chart to determine how many different outcomes of boys and girls are possible in a family with four children and then answer the questions that follow. Use B for boy and G for girl. Assume the first letter written represents the oldest child, the second letter is the second to oldest, etc.

4 Boys	3 Boys	2 Boys	1 Boy	0 Boys
BBBB	BBBG	BBGG	GGGB	GGGG

1. What is the total number of outcomes? How many outcomes have at least one girl? At least 3 girls? At most 2 boys?

2. What is the probability that a family with four children will have 2 girls and 2 boys?

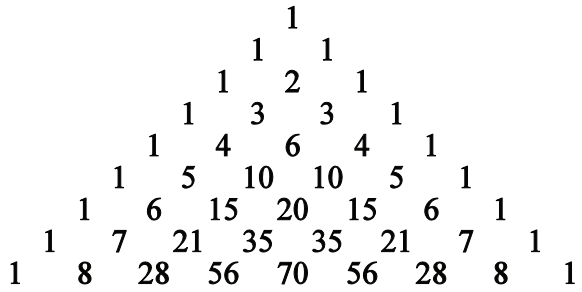
3. What is the probability of having no boys in a family with four children?

4. What is the probability that the oldest child is a boy? Explain how the table can be used to answer the question or explain how to calculate the answer.

5. What is the probability that the youngest two children are girls? Explain how the table can be used to answer the question or explain how to calculate the answer.

6. What is the probability that the second from the youngest is a girl, in a family of 2 boys and 2 girls? In a family of 3 boys and 1 girl?

7. Look at row 4 in Pascal's triangle. Note: The single 1 at the top is considered to be row 0.



   Write the total number of possible outcomes for **each** column on the last row of your boy/girl chart. What do you notice about those totals and the numbers in row four of Pascal's triangle?

8. What is true about the total of the numbers in row 4 and the total number of outcomes of boys and girls in a family of 4?

9. How many outcomes are there in the sample space for a family with 3 children? Compare your answer to the numbers in row 3 of Pascal's triangle.

10. Using Pascal's triangle, predict the number of different boy/girl outcomes in a family with 7 children.

11. Using Pascal's triangle and your answer to question 10, determine the probability of having exactly 3 boys in a family with 7 children.

12. What is the probability of having at least 3 boys in a family with 5 children?

13. Determine the probability of having exactly 4 boys in a family with 10 children. Explain your method.

14. Write 3 probability questions about a situation other than a family with children that can be determined using Pascal's triangle. Include the answers with your questions.