Bullseye!

ABOUT THIS LESSON

In this lesson, students use the areas of geometric figures as concrete models to develop probability concepts. Based on multiple scenarios involving dartboards, students calculate the theoretical probability for simple and compound events of landing in a specific area. They are also given a specific probability and are asked to determine the area that would give that probability.

This lesson focuses on determining probability while it also gives students practice in calculating the areas of triangles, rectangles, and trapezoids. It enhances student understanding of these standards by developing coherence and connections among a variety of mathematical concepts, skills, and practices.

OBJECTIVES

Students will

- calculate the area of geometric shapes.
- calculate the theoretical probability of simple events using the area of geometric shapes.
- calculate the probability of compound events.

LEVEL

Grade 7 in a unit on probability

MODULE/CONNECTION TO AP*

Probability

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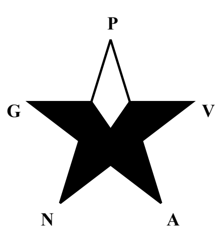
MODALITY

NMSI emphasizes using multiple representations to connect various approaches to a situation in order to increase student understanding. The lesson provides multiple strategies and models for using those representations indicated by the darkened points of the star to introduce, explore, and reinforce mathematical concepts and to enhance conceptual understanding.



P – Physical
V – Verbal
A – Analytical
N – Numerical
G – Graphical

COMMON CORE STATE STANDARDS FOR MATHEMATICAL CONTENT

This lesson addresses the following Common Core State Standards for Mathematical Content. The lesson requires that students recall and apply each of these standards rather than providing the initial introduction to the specific skill.

Targeted Standards

7.G.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.
See questions 1a-c, 3a-b, 4a, 4d-e

7.SP.8b Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
(b) Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event.
See questions 2e-h, 3f-g, 4b-c

Reinforced/Applied Standards

7.SP.5: Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.
See questions 1d, 2a-d, 3c-e

7.G.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
See questions 4a, 4d-e

COMMON CORE STATE STANDARDS FOR MATHEMATICAL PRACTICE

These standards describe a variety of instructional practices based on processes and proficiencies that are critical for mathematics instruction. NMSI incorporates these important processes and proficiencies to help students develop knowledge and understanding and to assist them in making important connections across grade levels. This lesson allows teachers to address the following Common Core State Standards for Mathematical Practice.

MP.6: Attend to precision.
In question 4, students understand their proportion in part (c) and the area of the circle formula in part (d) must be written as inequalities and answer both questions with a degree of precision appropriate to the problem.

FOUNDATIONAL SKILLS

The following skills lay the foundation for concepts included in this lesson:

- Calculate simple and compound probabilities
- Calculate area of rectangles, triangles, trapezoids, and circles

ASSESSMENTS

The following formative assessment is embedded in this lesson:

- Students engage in independent practice.

The following additional assessments are located on the NMSI website:

- Probability – 7th Grade Free Response Questions
- Probability – 7th Grade Multiple Choice Questions

MATERIALS AND RESOURCES

- Student Activity pages
- Calculators

TEACHING SUGGESTIONS

Before beginning the student activity pages, review how to determine the area of shaded regions and how to calculate simple and compound probability. It is reasonable to expect students to complete questions 1 and 2 without a calculator. In question 4, remind students that the circular target is not necessarily inscribed in the square. Unless stated, students should not make assumptions about the properties of geometric figures. Remind students to use the π button on their calculator when completing question 4.

You may wish to support this activity with TI-Nspire™ technology. See *Working with Fractions and Decimals* and *Using Special Keys in Calculations* in the NMSI TI-Nspire Skill Builders.

Suggested modifications for additional scaffolding include the following:

1 Modify the coordinate grid with the dimensions of each geometric figure labeled.

1d Provide the ratio in fractional form: $\dfrac{\text{area of isosceles triangle}}{\text{area of square}}$.

2e Provide the expression: $\left(\dfrac{\text{area of both triangles}}{\text{area of square}}\right)\left(\dfrac{\text{area of trapezoid}}{\text{area of square}}\right)$.

3b Set up the calculation of the area of one trapezoid.

4a Set up the area conversion using dimensional analysis.

4d Provide the inequality $\dfrac{A}{324 \text{ square inches}} \le 0.25$.

TEACHER PAGES

NMSI CONTENT PROGRESSION CHART

In the spirit of NMSI's goal to connect mathematics across grade levels, a Content Progression Chart for each module demonstrates how specific skills build and develop from sixth grade through pre-calculus in an accelerated program that enables students to take college-level courses in high school, using a faster pace to compress content. In this sequence, Grades 6, 7, 8, and Algebra 1 are compacted into three courses. Grade 6 includes all of the Grade 6 content and some of the content from Grade 7, Grade 7 contains the remainder of the Grade 7 content and some of the content from Grade 8, and Algebra 1 includes the remainder of the content from Grade 8 and all of the Algebra 1 content.

The complete Content Progression Chart for this module is provided on our website and at the beginning of the training manual. This portion of the chart illustrates how the skills included in this particular lesson develop as students advance through this accelerated course sequence.

6th Grade Skills/Objectives	7th Grade Skills/Objectives	Algebra 1 Skills/Objectives	Geometry Skills/Objectives	Algebra 2 Skills/Objectives	Pre-Calculus Skills/Objectives
Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.	Compute the probability of simple events and their complements.
Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements.	Compute the probability of compound events and their complements using the general addition and multiplication rules.	Compute the probability of compound events and their complements using the general addition and multiplication rules.	Compute the probability of compound events and their complements using the general addition and multiplication rules.
Compute probabilities based on the area of geometric figures.	Compute probabilities based on the area of geometric figures.	Compute probabilities based on the area of geometric figures.	Compute probabilities based on the area of geometric figures.	Compute probabilities based on the area of geometric figures.	Compute probabilities based on the area of geometric figures.

Bullseye!

Answers

1. a. $(10\text{ units})(10\text{ units}) = 100$ square units

b. Figure 1: Right Triangle: $\dfrac{(3\text{ units})(4\text{ units})}{2} = 6$ square units

Figure 2: Right Trapezoid: $\dfrac{1}{2}(2\text{ units})(2\text{ units} + 4\text{ units}) = 6$ square units

Figure 3: Rectangle: $(2\text{ units})(3\text{ units}) = 6$ square units

Figure 4: Isosceles Triangle: $\dfrac{(4\text{ units})(4\text{ units})}{2} = 8$ square units

c. 100 square units $-(6+8+6+6)$ square units $= 74$ square units

d. $\dfrac{8\text{ square units}}{100\text{ square units}} = \dfrac{8}{100}$ or 0.08 or 8%

2. a. $\dfrac{8\text{ square units}}{100\text{ square units}} = 0.08$ or 8%

b. $\dfrac{74\text{ square units}}{100\text{ square units}} = 0.74$ or 74%

c. $1 - \dfrac{74}{100} = \dfrac{26}{100}$ or 0.26 or 26%

d. $\dfrac{6\text{ square units} + 8\text{ square units}}{100\text{ square units}} = \dfrac{14}{100}$ or 0.14 or 14%

e. $\left(\dfrac{14}{100}\right)\left(\dfrac{6}{100}\right) = 0.84\%$

f. $\left(\dfrac{6}{100}\right)\left(\dfrac{74}{100}\right) = 4.44\%$

g. Answers may vary: Figures 1, 3, 4; Figures 2, 2, 4; Figures 4, 3, 3; Figures 1, 1, 4

h.

i. $\left(\dfrac{6}{100}\right)\left(\dfrac{8}{100}\right) = 0.0048$

ii. $\left(\dfrac{8}{100}\right)\left(\dfrac{8}{100}\right) = 0.0064$

TEACHER PAGES

iii. $\left(\frac{74}{100}\right)\left(\frac{74}{100}\right)=0.5476$

iv. $\left(\frac{12}{100}\right)\left(\frac{74}{100}\right)=.0888$

v. $\left(\frac{94}{100}\right)\left(\frac{94}{100}\right)=0.8836$

Probability of hitting neither dot in figure 3 has the highest probability at 0.8836.

3. a. (24 inches)(36 inches) = 864 square inches

b. $3\left[\frac{1}{2}(8\text{ inches})(8\text{ inches}+12\text{ inches})\right]=240\text{ square inches}$

c. $\frac{240\text{ square inches}}{864\text{ square inches}}\approx 0.27778\text{ or }27.778\%$

d. $\frac{80\text{ square inches}}{864\text{ square inches}}\approx 0.09259\text{ or }9.259\%$

e. $1-\frac{240\text{ square inches}}{864\text{ square inches}}\approx 0.72222\text{ or }72.222\%$

f. $\left(\frac{240}{864}\right)\left(\frac{80}{864}\right)\approx 0.02572\text{ or }2.572\%$

g. $\left(\frac{624}{864}\right)\left(\frac{624}{864}\right)\approx 0.5216\text{ or }52.16\%$

4. a. $\left(\frac{1.5\text{ feet}}{1}\right)\left(\frac{12\text{ inches}}{1\text{ foot}}\right)=18\text{ inches};(18\text{ inches})(18\text{ inches})=324\text{ square inches}$

b. $\left(\frac{\pi(6)^2}{324}\right)\left(\frac{\pi(6)^2}{324}\right)\left(\frac{\pi(6)^2}{324}\right)\approx 0.0425\text{ or }4.25\%$

c. The probability would be higher because the circle has a larger area but the area of the square is unchanged.

d. $\frac{A}{324\text{ square inches}}\leq 0.25;\ A\leq 81\text{ square inches}$

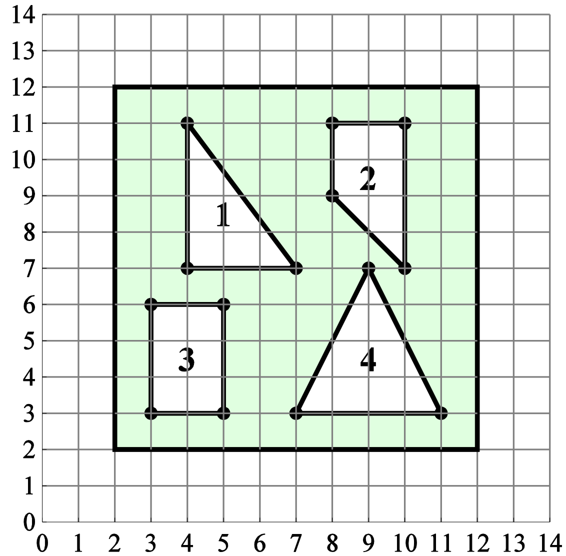
e. $A=\pi r^2;\ \pi r^2\leq 81\text{ square inches};\ r^2\approx 25.7831\text{ square inches};\ r\approx 5.078\text{ inches}$

The radius should be no more than 5 inches.

TEACHER PAGES

Bullseye!

1. Tasha drew several geometric figures in quadrant 1 of the coordinate grid in her math class.



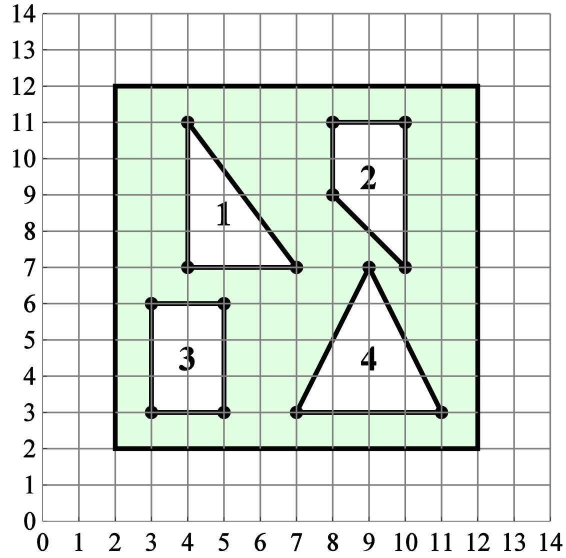
a. What is the area of the square formed by connecting the points (2, 2), (12, 2), (12, 12) and (2, 12)?

b. Classify each geometric figure inside the square and calculate the area for each shape.

c. What is the area of the shaded region which represents the area inside the square and outside of figures 1 – 4?

d. What is the ratio of the area of the isosceles triangle to the area of the square? Write your answer as a percent.

2. Tasha decided to use her drawing to determine the probability that a randomly selected point that lands inside the square would land in one of the figures 1 – 4. She asked her friend, Carissa, to close her eyes and randomly place a dot in the square.



a. What is the probability that her friend placed a dot in the isosceles triangle?

b. What is the probability that her friend placed a dot in the shaded region of the square?

c. What is the probability that her friend's dot is **not** in the shaded region of the square?

d. What is the probability that her friend placed a dot in either one of the triangles?

e. What is the probability that her friend placed one dot in either triangle and a second dot in the trapezoid?

f. What is the probability that her friend placed one dot in figure 2 and a second dot in the shaded area?

g. The probability of Tasha's friend placing one dot in figure one, then a second dot in figure 2, and then a third dot in figure 4 is 0.000288. Name two other combinations of placing dots that result in the same probability.

h. Which is the most likely outcome if Tasha's friend places one dot and then a second dot at random in the square?

i. The first dot in figure 1 and the second dot in figure 4.

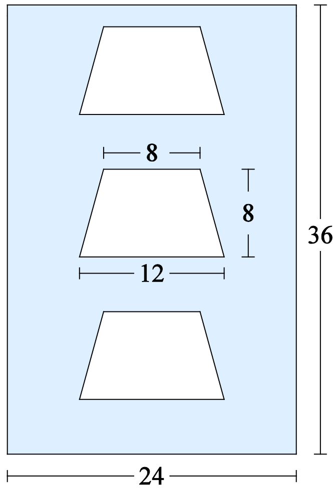
ii. Both dots in figure 4.

iii. Two dots in the shaded region.

iv. A dot in one of the triangles and a dot in the shaded region.

v. Neither dot in figure 3.

3. On a rectangular dartboard with a base of 24 inches and a height of 36 inches, three isosceles trapezoidal targets are drawn. Each trapezoid has a height of 8 inches and bases measuring 8 and 12 inches.



a. What is the area of the dartboard?

b. What is the total area of the three trapezoids?

c. If a dart hits the board, what is the probability that the dart lands in any of the three trapezoids?

d. If a dart hits the board, what is the probability that the dart lands in the middle trapezoid?

e. If a dart hits the board, what is the probability that the dart does not land in any of the trapezoids?

f. If two darts hit the board, what is the probability that a first dart hits any trapezoid and a second dart thrown hits the middle trapezoid?

g. If two darts hit the board, what is the probability that neither the first dart nor the second dart hit a trapezoid?

4. The 7th grade math class is designing a dartboard for the Fall Carnival. The base of the board will be a square with a circular center target. If three darts are thrown and each dart lands in the circular center target, the player wins a prize.

   a. If the side of the square is $1\frac{1}{2}$ feet, what is the area in square inches?

   b. If each dart hits the board, what is the probability that the player is a winner if the circular target has a diameter of 12 inches?

   c. If the circle in the center of the dart board was changed to have a radius of 9 inches. Would the probability of winning be higher or lower than the probability in (b)? Explain your reasoning.

   d. If the math class decides that throwing three darts takes too much time at the carnival and changes the rules to where each contestant throws only one dart. If the dart is thrown and lands in the circular center target, the player wins the prize. They want the probability of winning to be less than or equal to 25%, what is the maximum area of the circle?

   e. Based on your answer in (d), what is a reasonable measurement for the radius of the circular target? Round your answer to the nearest whole number.