



# Chi Critters

## Analyzing Populations Affected by Population-Limiting Factors

### MATERIALS AND RESOURCES

EACH GROUP	TEACHER
Adirondack mice bag	56 bags, small pill
calculator	14 bags, zipper-lock, gallon
copy of assembled grid	2 pkg beans, navy
copy of site description cards	2 pkg beans, pinto
die	2 pkg beans, red
tape, clear	2 pkg beans, white
scissors	14 copies of Adirondack mice descriptions (sets of 4)
	14 copies of Adirondack mice pictures (sets of 4)

### ABOUT THIS LESSON

Students will be guided through various activities to understand the calculations and purpose of the chi-squared test. A basic understanding of using a null hypothesis will also be established. Students will apply their understanding of the principle to a variety of scenarios.

### OBJECTIVES

Students will:

- Develop and test a null hypothesis using the chi-squared test
- Understand the significance of using the chi-squared test to validate or discredit the null hypothesis
- Analyze the causes of population fluctuations in four mice populations that are native to the Adirondack Mountains
- Utilize the chi-squared equation to determine if the size of the population change is as a result of density-dependent or density-independent factors in the region

### LEVEL

Biology

## NEXT GENERATION SCIENCE STANDARDS

DEVELOPING AND  
USING MODELSANALYZING AND  
INTERPRETING DATA

USING MATHEMATICS



PATTERNS

SCALE, PROPORTION,  
AND QUANTITYSTABILITY  
AND CHANGE

LS2: ECOSYSTEMS

## LESSON CONSUMABLES

Item 1 – Key to Mice

Item 2 – Grid Piece (1/9)

Item 3 – Grid Piece (2/9)

Item 4 – Grid Piece (3/9)

Item 5 – Grid Piece (4/9)

Item 6 – Grid Piece (5/9)

Item 7 – Grid Piece (6/9)

Item 8 – Grid Piece (7/9)

Item 9 – Grid Piece (8/9)

Item 10 – Grid Piece (9/9)

Item 11 – Species Description Cards (1/2)

Item 12 – Species Description Cards (2/2)

Item 13 – Site Description Cards

## ASSESSMENTS

The following types of formative assessments are embedded in this lesson:

- Assessment of prior knowledge by checking pre-lab answers
- Guided questioning during chi-squared calculations
- Visual observation of student performance during activity
- Observations of data collected in data tables throughout the activity
- Completion and discussion of conclusion questions

## RESOURCES

“Beech Nuts, Mice and Bears,” Stacy McNulty, *Adirondack Almanack*, [www.adirondackalmanack.com/2012/08/stacy-mcnulty-beech-nuts-mice-and-bears.html](http://www.adirondackalmanack.com/2012/08/stacy-mcnulty-beech-nuts-mice-and-bears.html)

“Is another Adirondack fire disaster on the way?” *Adirondack Almanack*, [www.adirondackalmanack.com/2007/10/is-another-adirondack-fire-disaster-on-the-way.html](http://www.adirondackalmanack.com/2007/10/is-another-adirondack-fire-disaster-on-the-way.html)

“Adirondacks see rise in mouse population,” *USA Today*, [http://usatoday30.usatoday.com/tech/science/2007-10-03-1871751263\\_x.htm](http://usatoday30.usatoday.com/tech/science/2007-10-03-1871751263_x.htm)

[www.esf.edu/aec/adks/mammals/deer\\_mouse.htm](http://www.esf.edu/aec/adks/mammals/deer_mouse.htm)

[www.esf.edu/aec/adks/mammals/meadow\\_jumping.htm](http://www.esf.edu/aec/adks/mammals/meadow_jumping.htm)

[www.esf.edu/aec/adks/mammals/whitefooted\\_mouse.htm](http://www.esf.edu/aec/adks/mammals/whitefooted_mouse.htm)

[www.esf.edu/aec/adks/mammals/woodland\\_jumping.htm](http://www.esf.edu/aec/adks/mammals/woodland_jumping.htm)

[www.fcps.edu/islandcreekes/ecology/white-footed\\_mouse.htm](http://www.fcps.edu/islandcreekes/ecology/white-footed_mouse.htm)

[www.adkforum.com/showthread.php?t=10972](http://www.adkforum.com/showthread.php?t=10972)

[www.medcalc.org/manual/chi-square-table.php](http://www.medcalc.org/manual/chi-square-table.php)

**COMMON CORE STATE STANDARDS****(LITERACY) RST.9-10.3**

Follow precisely a multistep procedure when carrying out experiments, taking measurements, or performing technical tasks, attending to special cases or exceptions defined in the text.

**(LITERACY) RST.9-10.7**

Translate quantitative or technical information expressed in words in a text into visual form (e.g., a table or chart) and translate information expressed visually or mathematically (e.g., in an equation) into words.

**(MATH) S-ID.4**

Use data from a sample survey to estimate a population mean or proportion; develop a margin of error though the use of simulation models for random sampling.

**ACKNOWLEDGEMENTS**

Image of deer mouse (*Peromyscus maniculatus*) from Wikimedia Commons. Public domain.

Image of meadow jumping mouse (*Zapus hudsonius*) from Wikimedia Commons. Public domain.

Image of white-foot mouse (*Peromyscus leucopus*) from Centers for Disease Control and Prevention (CDC). Public domain.

Image of woodland jumping mouse (*Napaeozapus insignis*) used under the Creative Commons Attribution-Share Alike 3.0 Unported license. Copyright © 2008 Wikimedia user Gordon E. Robertson.

**CONNECTIONS TO AP\***

- A.4 Biological evolution is supported by scientific evidence from many disciplines, including mathematics.



- A.5 Communities are composed of populations of organisms that interact in complex ways.
- A.6 Interactions among living systems and with their environment result in the movement of matter and energy.

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## TEACHING SUGGESTIONS

In this lesson, students will use a random sampling technique to gather the population sizes of four different mice populations that inhabit the Adirondacks. The chi-squared test will be performed on mouse populations to determine if the size of the population change is a result of density-dependent or density-independent factors in the region.

### PART I: CHI-SQUARED TEST

Part I gives the students the prior knowledge required to complete the investigation. It is important before you perform Part II that students understand the purpose and calculations involved with the chi-squared equation:

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}} \quad (\text{Eq. A})$$

This portion of the lesson is simply designed so that students have this basic understanding and can utilize the equation with ease.

You can either purchase 30 dice for use in the Pre-Lab Exercises or utilize a dice roller app. If you only have 30 dice, perform the investigation as a class. This would be a good idea if you want to be certain that all of the students are utilizing the same data for their first time through chi-squared problems. On the other hand, having students in groups each generate their own totals would give the class a variety of chi-squared values, which would be a good source of discussion.

Depending on what you have done with calculations previously in the school year, you must decide how to handle the small values that students will get while solving these problems. In truth, rounding numbers at all when performing chi-squared analysis actually makes the value more invalid.

Because this activity is set up in a simplified manner, students can actually calculate the values going down each column. By working down the column, students do not need to round the numbers until the next-to-last step ( $d/e$ ).

When deciding how far to round, there are basically two options that work best with these equations. Option 1 is to have students keep all numbers to the right of the decimal. Because these numbers are considered counted numbers, each number to the right of the decimal is of value. In this case, you would have students round their final answers to include four significant figures. This method will yield the most accurate results and chi value.

Option 2 is to have students set the FLOAT value to 4 if using a graphing calculator. This will cause the calculator to automatically round the values for students to four places past the decimal. To do this, students should press “MODE,” arrow over to “4,” and press “ENTER” twice.

Guide students through writing a “null hypothesis.” Assist students in filling in the rows labeled “Observed” and “Percent Chance.” Once students have filled in the row for “Percent Chance,” they will see that they have already made a decision that there is the same number of chances for them to roll each of the numbers on the dice. Therefore, the transition to writing the null hypothesis should be an easy one because it can be assumed it is equally likely that a student would roll each of the other numbers.

Work with students to generate their null hypothesis assuming there is a good chance that if they roll the dice, the observed number of times each number will be rolled will not be significantly different than what is expected. Thus, the null hypothesis would state: “There is no significant difference between the observed and expected frequencies.”

## TEACHING SUGGESTIONS (CONTINUED)

Introduce students to the complete equation (Equation A). Explain to them that we are using the same equation, but you wanted to present it to them in an easier format. The chi-squared equation has been broken down into pieces as shown in the data table. Students start off by using the table with each step of the equations listed in each row. After calculating chi-squared in this step-by-step manner, students understand and eventually memorize the equation.

The step in the lesson that says “Record in the data table the total number of rolls that you made as ‘30’” is the most frequent place students tend to make a mistake when performing calculations using the chi-squared test. It is very important that students remember to total this row. Whether totaling up the number of organisms, the number of candies, or the number of offspring, be certain that they total this row.

Students struggle when trying to determine the expected amounts. Some struggle with proportions whereas others struggle with percentages. The biggest mistake with this step is that students forget to calculate the expected value based on the total number.

For example, using the expected percentages, calculate the expected number of times that each number should be rolled.

$$\frac{16.67}{100} = \frac{x}{30} \Rightarrow x = 5$$

Students will plug in the 16.67 as the expected number rather than the 5.

From this point on, guide students through each step on each row. The equation for what they do on each row is presented on the table. The final row involves

the symbol  $\Sigma$ , which means “sum of.” Add all of the numbers in the row labeled  $d^2/e$ .

Assist students in calculating the degrees of freedom. To calculate the degree of freedom, subtract 1 from the number of categories. In this case, there are 6 numbers on a die and therefore there are 6 categories. The degrees of freedom will be 5.

Guide students through using the chi-squared distribution table:

- Locate the row that contains the correct degree of freedom.
- Skim across the row to find the chi-squared value that was calculated.
- Once they locate the area where their value would be, find the next smallest number. Go to the top of the column and find the probability. The probability value is the probability that a deviation as great or greater than each chi-squared value would occur by chance.

Many biologists agree that chi-squared values greater than 0.05 or 5% are not statistically significant. If the chi-squared value falls under 0.05, then the null hypothesis would be rejected.

For example, because the degree of freedom is 5 and the chi-squared value is 2.8, then your probability would be expressed as

$$p < 0.750$$

As you can see, this number falls to the left of the 0.05 critical value and thus is in the “accept hypothesis” range.

## TEACHING SUGGESTIONS (CONTINUED)

## PART II: POPULATION SAMPLING

Introduce the concept of *population sampling* to students. Explain to them that there are different methods of sampling the population, and in this activity they will use the method known as *random sampling*. For reference, there are practice problems with an explanation in the middle grades lesson, “Field of Beans,” on random sampling.

Divide students into groups of three at each station. Each station should include a bag of beans, mouse description cards, and the grid paper provided in the lesson.

Have them read over the procedure and begin sampling the population. They will count only the beans that have landed inside the squares. If a bean has landed on the line, you must decide as a class if the bean will count as being inside or outside the box. Whatever decision is made, it must be consistent as a class.

Once they have completed the counting of the organisms, have students generate a null hypothesis. We are assuming that their technique as well as the random sampling technique is very accurate, therefore the null hypothesis could be: “There will be no significance difference between the actual sizes of the four mice populations and the estimated population sizes.”

Provide students with the actual total number of organisms that they should have counted. Each population should have counted 200 organisms.

Have students use the chi-squared test to determine whether their null hypothesis is supported or rejected. It will most likely fall in the “Rejected” portion of the chi-squared distribution table. Brainstorm with students some possible sources of error for their data. Some explanations are:

- Random sampling is only an estimate of the population size. It does not count every single organism present.
- There were only 10 samples taken from the entire grid. Perhaps had more samples been taken, the data would be more accurate.
- The scale of the beans to the grid was not proportional.

## PREPARING FOR PART II OF THE ACTIVITY

Assemble the eight bags of beans prior to the activity. Once you have made the bags, you can reuse them each year.

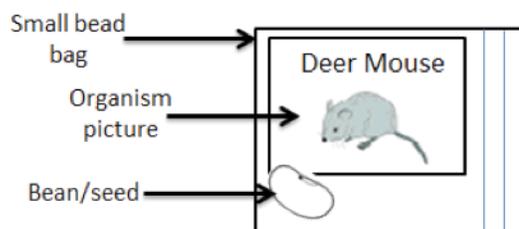


Figure A. Small bead bag and contents

Each gallon bag should contain:

- 1 small bead bag containing a picture of each mouse and the corresponding bean.
- 200 of each of the four types of beans = 800 total beans
- 1 piece of folded grid paper
- Mouse Description cards

Key to mice:

- Cut out each of the mouse grid squares.
- Place each in a small bead bag with its corresponding bean.

## TEACHING SUGGESTIONS (CONTINUED)

## ASSEMBLING THE GRIDS

1. Cut out each of the pieces of grid paper.
2. Tape them together as directed by the labels. The labels should combine in the center in order to make a square (Figure B).

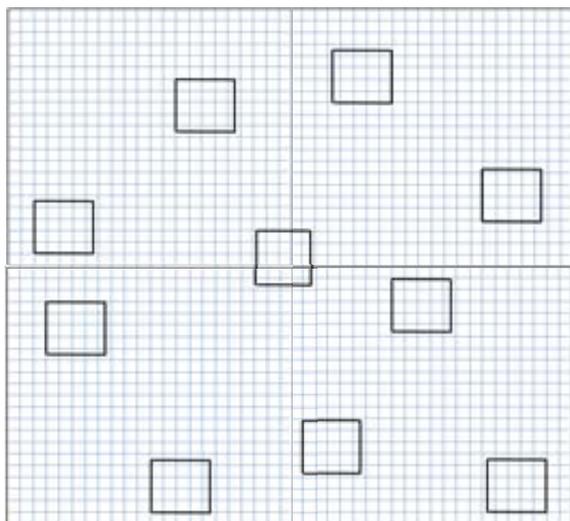


Figure B. Assembled grids

3. Fold up the edges of the paper to prevent the beans from rolling off the paper (Figure C). This step is not required but can help in preventing the loss of beans.



Figure C. Rolled-up edges

## GOING FURTHER

Now that students have seen there are errors in this technique, they could possibly research other techniques used today by ecologists to calculate the size of populations.

## PART III: ADIRONDACK MICE

In this portion of the activity, students will take existing data collected from three regions in the Adirondack Mountains over a three year time period. At this point, students should be able to generate their own data table using the steps to solve a chi-squared test. They should be able to work the equation without using the data table at all.

All three sites started with the same number of mice. Assuming that the population has maintained equilibrium each successive year, the same percentage is expected.

Students will continue to use the expected percentages when analyzing the Year 3 data because the balanced numbers would be at 25%.

Some assistance may be required for students to develop their first chi-squared chart if they are still not comfortable with the equation without the chart.

After the students complete Table 4, distribute the “Site Description” cards to each group. Students will need to determine which of the descriptions would match the data from each of the sites.

**PRE-LAB EXERCISES**

1. If  $x = 5.0$ , calculate  $x^2$ .

$$x^2 = 25$$

2. If  $y = 13.2$ , calculate  $y^2$ .

$$y^2 = 174.24$$

3. Solve the following equations for  $x$ .

a.  $\frac{12}{100} = \frac{x}{70}$

$$x = 8.4$$

b.  $\frac{64}{100} = \frac{x}{70}$

$$x = 44.8$$

4. In a container of candies, 13% of them are supposed to be red. If there were 50 candies in the bag, how many are expected to be red?

$$0.13 \times 50 = 6.5 \text{ should be red}$$

5. According to the candy company, each bag should contain 20% of each color (red, blue, green, brown, and yellow). If you purchase a bag of candy that has 120 candies in it, how many of each color should be in the bag?

24 of each color

6. Predict the likelihood that a 3 will appear when rolling a six-sided die.

1 out of 6

7. Calculate the percent chance that a 3 will appear out of all of the numbers on a six-sided die.

1 out of 6 = 16.67%

**DATA AND OBSERVATIONS**

**PART I: CHI-SQUARED TEST**

Degrees of freedom: Number of categories – 1 = 5

**NULL HYPOTHESIS**

There is no significant difference between the observed and expected frequencies of the die rolls.

Table 2. Chi-Squared Test for Rolling a Die							
	1	2	3	4	5	6	Total
Percent chance (%)	16.67	16.67	16.67	16.67	16.67	16.67	
Expected (e)	5	5	5	5	5	5	30
Observed (o)	7	4	5	3	4	7	30
Difference ( $d = o - e$ )	2	-1	0	-2	-1	2	
Difference squared ( $d^2$ )	4	1	0	4	1	4	
$d^2 / e$	0.8	0.2	0.0	0.8	0.2	0.8	
$\chi^2 = \Sigma(d^2 / e)$	2.8						

ANSWER KEY

## DATA AND OBSERVATIONS (CONTINUED)

## PART II: POPULATION SAMPLING

## NULL HYPOTHESIS

There is no significant difference between the observed and expected frequencies of the mouse populations.

Animal	Total in 10 Sample Sites	Average Counted (Total / 10)	Total Population Size	
			Estimated (Average $\times$ 100)	Actual (From Teacher)
Deer mouse	35	3.5	350	200
White-footed mouse	42	4.2	420	200
Woodland jumping mouse	37	3.7	370	200
Meadow jumping mouse	30	3.0	300	200

	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed ( <i>o</i> )	350	420	370	300	1440
Expected ( <i>e</i> )	360	360	360	360	1440
Difference $d = (o - e)$	-10	60	10	-60	
Difference squared ( $d^2$ )	100	3600	100	3600	
$d^2 / e$	0.2778	10.0000	0.2778	10.0000	
$\chi^2 = \Sigma(d^2 / e)$	20.5556				

## DATA AND OBSERVATIONS (CONTINUED)

## PART III: ADIRONDACK MICE

Table A. Adirondack Mice, Site 1, Year 1 to Year 2					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed ( <i>o</i> )	198	219	190	202	800
Expected ( <i>e</i> )	200	200	200	200	800
Difference, $d = (o - e)$	-2	10	-10	2	
Difference squared ( $d^2$ )	4	100	100	4	
$d^2 / e$	0.02	0.50	0.50	0.02	
$\chi^2 = \Sigma(d^2 / e)$	1.04				

Table B. Adirondack Mice, Site 1, Year 1 to Year 3					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed ( <i>o</i> )	205	199	203	193	800
Expected ( <i>e</i> )	200	200	200	200	800
Difference, $d = (o - e)$	5	-1	3	-7	
Difference squared, ( $d^2$ )	25.0000	1.0000	9.0000	49.0000	
$d^2 / e$	0.1250	0.0050	0.0450	0.2450	
$\chi^2 = \Sigma(d^2 / e)$	0.4200				

Table C. Adirondack Mice, Site 2, Year 1 to Year 2					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed ( <i>o</i> )	70	80	107	115	372
Expected ( <i>e</i> )	93	93	93	93	372
Difference, $d = (o - e)$	-23	-13	14	22	
Difference squared ( $d^2$ )	529.0000	169.0000	196.0000	484.0000	
$d^2 / e$	5.6882	1.8172	2.1075	5.2043	
$\chi^2 = \Sigma(d^2 / e)$	14.8172				

ANSWER KEY

DATA AND OBSERVATIONS (CONTINUED)

Table D. Adirondack Mice, Site 2, Year 1 to Year 3					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed (o)	90	100	130	140	460
Expected (e)	115	115	115	115	460
Difference, $d = (o - e)$	-25	-15	15	25	
Difference squared ( $d^2$ )	625	225	225	625	
$d^2 / e$	5.4348	1.9565	1.9565	5.4348	
$\chi^2 = \Sigma(d^2 / e)$	14.7626				

Table E. Adirondack Mice, Site 3, Year 1 to Year 2					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed (o)	180	180	90	175	625
Expected (e)	156.25	156.25	156.25	156.25	625
Difference, $d = (o - e)$	23.75	23.75	-66.25	18.75	
Difference squared ( $d^2$ )	564.0625	564.0625	4389.0625	351.5625	
$d^2 / e$	3.6100	3.6100	28.0900	2.2500	
$\chi^2 = \Sigma(d^2 / e)$	37.5600				

Table F. Adirondack Mice, Site 3, Year 1 to Year 3					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)	25	25	25	25	
Observed (o)	185	190	125	205	705
Expected (e)	176.25	176.25	176.25	176.25	705
Difference, $d = (o - e)$	8.75	13.75	-51.25	28.75	
Difference squared ( $d^2$ )	76.5625	189.0625	2626.5625	826.5625	
$d^2 / e$	0.4344	1.0727	14.9025	4.6897	
$\chi^2 = \Sigma(d^2 / e)$	21.0993				

ANSWER KEY

## DATA AND OBSERVATIONS (CONTINUED)

Table 5. Chi-Squared Test on Adirondack Mice Populations		
Site	Calculations Between...	Chi-Squared Value
1	Year 1 to Year 2	1.4000
	Year 1 to Year 3	0.4200
2	Year 1 to Year 2	14.8172
	Year 1 to Year 3	14.7826
3	Year 1 to Year 2	37.5600
	Year 1 to Year 3	21.0993

## ANALYSIS

## PART I: CHI-SQUARED TEST

- Using Table 2 and the chi-squared result from Part I, write a conclusion statement summarizing your results. Be sure to restate your null hypothesis, chi-squared value, and statistical probability when making this statement.

The null hypothesis was “There is no significant difference between the observed and expected frequencies of the die rolls.” The chi-squared value determined was 2.8 with a statistical probability of 0.90.

## PART II: POPULATION SAMPLING

- Was your null hypothesis supported or rejected by your data? Justify your answer.

The chi-squared value was 20.5556 with a degree of freedom of 3. The probably would be  $p < 0.005$ , which means that our null hypothesis is rejected. The chi-squared value is so low that it is not even on the chart.

- Sources of error are typically a part of any data collection. Name at least three things that you can see as being sources of error that would have contributed to collecting not the most accurate data.

Three sources of possible error are:

- Random sampling is only an estimate of the population size. It does not count every single organism present.
- There were only 10 samples taken from the entire grid. Perhaps had more samples been taken, the data would have been more accurate.
- The scale of the beans to the grid was not proportional. The grid should have been significantly larger than it was designed.

**ANALYSIS (CONTINUED)****PART III: ADIRONDACK MICE**

1. Chi-squared calculations made in Table 5 were all based on the Year 1 percentages. What other data could be used to determine the chi-squared values? Predict how the chi-squared values would be impacted as a result of your suggestion.

The chi-squared test could use the Year 2 data and the percentages that were present in Year 2.

2. Explain why you selected each of the Site Descriptions to correspond with each of the three sites.

Site 1 corresponds with Site Description 4. The chi-squared values were low (1.04 and 0.4200), which are both greater than 0.25. Because the chi-squared value was within the accepted hypothesis range, there was not an external factor that caused a population fluctuation.

Site 2 corresponds with Site Description 1. As a result of the extremely low temperature, the populations of mice across the board decreased. Even with the drastic decrease, the chi-squared values were 14.8172 and 14.7826.

Site 3 corresponds with Site Description 3. The huge chi-squared value is a result of an increase or decrease in one of the four populations. The meadow jumping mouse is the only one of the four mice that does not cache its food. As a result of a decrease in precipitation, the trees most likely did not produce an abundant number of seeds. Proof of this statement is evident in the fact that the bears are raiding campgrounds also as a result of a lack of food.

3. List and describe two density-dependent limiting factors and explain how each would limit population growth.

Density-dependent factors include a reduction in food supply, which would cause a decrease in the population that relies on that food source. An increase in predators would decrease the prey population. An increase in birth rates with a decrease in death rates would cause an increase in total population numbers, which would in turn decrease the food supply.

4. List and describe two density-independent limiting factors and explain how each would limit population growth.

A density-independent factor that can reduce population sizes would be a severe storm that causes flooding, sudden unpredictable cold spells, earthquakes, volcanoes, an plague, or a catastrophic meteorite impact.

5. Which site can you conclude was impacted by a density-dependent factor? Justify your answer.

Site 3 was impacted by a density-dependent factor, specifically a reduced food source of the meadow jumping mouse.

6. Which site can you conclude was impacted by a density-independent factor? Justify your answer.

Site 2 was impacted by a density-independent factor such as the record-breaking decrease in temperature. This would have wiped out many populations of animals.

## CONCLUSION QUESTIONS

- In 2012, a salmonella outbreak took place that caused grocery store shelves to sit empty of their peanut butter containers. Scientists at the Center for Disease Control and Prevention (CDC) utilized chi-squared equations to determine that there was in fact an outbreak and not a singular strange occurrence. Explain how the chi-squared test could be used to support their declaration of an outbreak.
- Two people that are carriers for sickle cell anemia have children even though doctors told them of their concern that one fourth of their children would be born with the same condition. After their 11 children were born, genetic screening provided them with the genotypes of each child. Three children were homozygous dominant, five were carriers, and the remaining children had sickle cell anemia. Determine the chi-squared value for this family and state whether the doctor's suspicion had any validity.

The scientists at the CDC would utilize chi-squared values to determine the outbreaks. On a normal day, the chi-squared value would be within the acceptable range. In the event of an outbreak, the value would fall in the rejected range if not off of the chi-squared distribution table altogether. This would immediately alert the scientists to a larger issue or outbreak that must be addressed.

The chi-squared value is 0.0455 and has a probability statistic of 0.90 (Table G). As a result, the doctor's suspicions were valid based on the offspring that the couple produced.

Table G. Children with Sickle Cell Anemia				
	Normal	Carriers	Sickle Cell Anemia	Totals
Percent chance (%)	25	50	25	
Observed (o)	3	5	3	11
Expected (e)	2.75	5.50	2.75	11
Difference, $d = (o - e)$	0.25	0.00	0.25	
Difference squared ( $d^2$ )	0.0625	0.0000	0.0625	
$d^2 / e$	0.0227	0.0000	0.0227	
$\chi^2 = \Sigma(d^2 / e)$	0.0455			

Table H. Colors of Corn Snakes					
	Normal	Anerhythmic (Grey)	Amelanistic (Red)	Snow	Totals
Percent chance (%)	56.25	18.75	18.75	6.25	
Observed (o)	4	3	2	2	11
Expected (e)	6.1875	2.0636	2.0636	.6875	11
Difference, $d = (o - e)$	-2.1875	0.9364	-0.0636	1.3125	
Difference squared ( $d^2$ )	4.7852	0.8768	0.0040	1.7227	
$d^2 / e$	0.7734	0.4249	0.0020	2.5057	
$\chi^2 = \Sigma(d^2 / e)$	3.7059				

**CONCLUSION QUESTIONS (CONTINUED)**

3. Mrs. Lowery bred her corn snakes so she could have baby snakes. The parents or P1 generation are “double hets,” which means that they are heterozygous for both traits that determine the color in corn snakes. In the F1 generation, four normal-colored, three grey, two red, and two snow-colored corn snakes hatched.

a. State the null hypothesis.

There should be no significant difference between the predicted numbers and the actual numbers of each color of corn snake that is produces.

b. Determine the chi-squared value and the validity to the null hypothesis.

The chi-squared value was 3.7059 with a statistical probability of 0.5 (Table H). Even though the probability is lower than preferred, it is still within the acceptable range and the null hypothesis would be valid.

4. The local hobby shop sells rock collections on a daily basis. In each collection there are generally the same types and quantities of rock, and there are usually 100 rocks in each jumbo collection. Each collection made by the hobby shop is usually about 20% of each type of rock.

When Jenny returned home with her new jumbo rock collection, she immediately started sorting and counting her new rocks. She counted each of the following: pyrite 19, quartz 18, halite 21, granite 20, and limestone 22.

a. State the null hypothesis.

There is no significant difference between the observed and expected frequencies

b. Calculate the chi-squared value for Jenny’s collection.

See Table I.

c. Was Jenny’s collection typical of the collections normally sold by the hobby shop?

Yes

d. Would your null hypothesis be accepted or rejected? Justify your answer.

The null hypothesis would be accepted due to the fact that the chi-squared value is above the 5% probability. It can be determined that the collection is typical to the normal collections.

ANSWER KEY

**Table I. Distribution of Rock Collection**

	Pyrite	Quartz	Halite	Granite	Limestone	Totals
Percent chance (%)	20	20	20	20	20	
Observed (o)	19	18	21	20	22	100
Expected (e)	20	20	20	20	20	100
Difference, $d = (o - e)$	-1	-2	1	0	2	
Difference squared ( $d^2$ )	1	4	1	0	4	
$d^2 / e$	0.05	0.20	0.05	0.00	0.20	
$\chi^2 = \Sigma(d^2 / e)$						0.50

## CONCLUSION QUESTIONS (CONTINUED)

5. In a given population, the evolution of genes can be evaluated using the Hardy-Weinberg equation. The Hardy-Weinberg equations were applied to the inheritance of hitchhiker’s thumb, a recessive condition.

A population was sampled and it was determined that 24% of the population has this recessive condition. People with the dominant phenotype have straight thumbs.

- a. Use the Hardy-Weinberg equation to calculate the “expected” frequency of each genotype.

The expected percentages are homozygous dominant = 0.26, heterozygous = 0.4998, and homozygous recessive = 0.24

- b. Each individual in the population was identified and labeled according to their genotype. In some cases, the parents of individuals had to be contacted to verify the label. In a population of 400 people, 160 were homozygous dominant, 144 were

heterozygous, and 96 were homozygous recessive. Using the calculations from the previous question in addition to these observed numbers, calculate the chi-squared value for this data.

The chi-squared value was 45.8338 with a probability statistic of less than 0.001, which is completely off the chart (Table J). As a result of the observed numbers being significantly different from the expected numbers, there must be an “external factor” that has impacted the population. In this case, one would say that the population is evolving.

This could be as a result of one of the mechanisms of change addressed in the Hardy-Weinberg equilibrium. The five mechanisms of change are gene flow, mutations, natural selection, non-random mating, or genetic drift, with natural selection being the least likely in this case.

Table J. Hitchhiker’s Thumb in a Population

	Homozygous Dominant	Heterozygous	Homozygous Recessive	Totals
Percent chance (%)	26	49.98	24	
Observed ( <i>o</i> )	160	144	96	400
Expected ( <i>e</i> )	104	200	96	400
Difference, $d = (o - e)$	56	-56	0	
Difference squared ( $d^2$ )	3136	3136	0	
$d^2 / e$	30.1538	15.6800	0.0000	
$\chi^2 = \Sigma(d^2 / e)$	45.8338			

## GOING FURTHER

Answers will vary.



NATIONAL  
MATH + SCIENCE  
INITIATIVE

#### MATERIALS

*Adirondack mice bag*

*calculator*

*copy of assembled grid*

*copy of site description  
cards*

*die*

*tape, clear*

*scissors*

# Chi Critters

## Analyzing Populations Affected by Population-Limiting Factors

In 1900, Karl Pearson first investigated the properties of the chi-squared test. The **chi-squared test** is the best known of the statistical procedures. It has the ability to test a null hypothesis that states that the frequency distribution of events observed in a sample are consistent with the expected distribution.

The chi-square equation is given in Equation 1:

$$\chi^2 = \sum \frac{(\text{observed}-\text{expected})^2}{\text{expected}} \quad (\text{Eq. 1})$$

The chi-squared test along with the null hypothesis can be used to analyze data that is gathered in many fields of science. The statistical procedure can be used when analyzing the outcome of offspring based on their predicted likelihoods.

The chi-squared test can also be used to determine if a population of organisms is fluctuating or maintaining equilibrium. It is this latter use of the chi-squared test that this activity will utilize.

The Adirondack Mountains in upstate New York are home to a wide variety of small mammals. Four mouse species will be counted and analyzed that inhabit the forests in this mountain range. The deer mouse and white-footed mouse are extremely similar in habitat and appearance. Slight differences between the two can be seen by ecologists that study the small mammals.

The meadow jumping mouse and the woodland jumping mouse are another pair of less similar mice. Coloring of the body, storage of food, and activity patterns are just a few differences between the jumping mice.

After a chi-squared test is performed on the populations, it will be determined which factors are likely impacting the sizes of the populations. Factors that impact the mouse populations can be one of two types, *density-dependent* factors or *density-independent* factors.

An increase in population size reduces the available resources, limiting a population's growth. The population size will then decrease as a result of a lack of food supply. The population size could also decrease as a result of increased predation or a decrease in birth rates. This increase or decrease as a result of population density would be known as a density-dependent factor.

A density-independent factor typically does not impact a few populations but impacts many populations drastically at one time. Some examples of factors that reduce population sizes but are independent of density would be a severe storm that causes flooding, cold spells, earthquakes, volcanoes, or a catastrophic meteorite impact.

### **PURPOSE**

In this activity, you will develop and test a null hypothesis using the chi-squared test and understand the significance of using the chi-squared equation to validate the null hypothesis. You will also investigate several density-dependent and density-independent population-limiting factors.

**PRE-LAB EXERCISES**

1. If  $x = 5.0$ , calculate  $x^2$ .

2. If  $y = 13.2$ , calculate  $y^2$ .

3. Solve the following equations for  $x$ .

a.  $\frac{12}{100} = \frac{x}{70}$

b.  $\frac{64}{100} = \frac{x}{70}$



**PROCEDURE****PART I: CHI-SQUARED TEST**

As a group, you will roll a die 30 times to compare how often each of the numbers is rolled to the number of times that you actually roll the die.

1. Generate the null hypothesis for this activity and record it on your student answer page.
2. Roll a die 30 times. Record the number that you roll each time in Table 1 in the row labeled “Observed.”
3. Record in the data table the total number of rolls that you made as “30” under the “Total” column.
4. Based on the practice problems in the Pre-Lab Exercises, fill in the percent chance that you would roll each of the numbers on the die in the row labeled “Percent Chance” in Table 2.
5. Your teacher will guide you through the calculations that are required for the next several steps.

**PROCEDURE (CONTINUED)****PART II: POPULATION SAMPLING**

You will be given a bag of beans from your teacher. This bag of beans represents four different types of mice and their population sizes. You will use a random sampling technique to estimate the total number of mice in each population as described in the following steps.

1. Take the small bead bag out of the gallon-sized bag and read the small paper inside the small bead bag.
2. When directed, pour out the bag of beans onto the grid paper. Spread out the beans as evenly as possible. The scale for the cutouts is 1:1, so you should have a 50 cm × 50 cm grid when assembled.
3. With your group, identify which mouse is represented by each type of bean. Read over the mouse description cards for each mouse that is located at your station. As a group, discuss the various mice that are represented in your bag.
4. You should have noticed when you spread out the beans that there were ten sample squares spread throughout the grid. Count the beans in each of the ten sample sites. Record these totals in Table 3.
5. Once you have counted all of the mice in each of the ten sample sites, find the average number of each type of mouse and record this value in Table 3.
6. Remembering that each sample site is 1/100<sup>th</sup> the size of the entire field, use your average to estimate the total number of organisms.
7. Your teacher will now tell you the actual number for each type of mouse that you should have estimated. Before they do this, generate a null hypothesis for your data and record it on your student answer pages.
8. Using your data as well as the data provided for you by your teacher, perform a chi-squared test on the data. Utilize Table 4 to complete this step.

**PROCEDURE (CONTINUED)****PART III: ADIRONDACK MICE**

Data was collected on the four mice populations in three different locations of the Adirondack Mountains, shown in Table 1. Ecologists in the Adirondacks analyze this type of data on a regular basis to determine the impact of various factors on the mice populations as well as many others.

The data from each of the sites in this activity was collected over the past 25 years from different locations. Descriptions were written by ecologists at the time for each of the sites and have been recorded on the cards labeled “Site Description.”

1. Perform a chi-squared test from Year 1 to Year 2, and then from Year 1 to Year 3, on each of the sites. As you perform each chi-squared test, complete Table 5 on your student answer pages.
2. Using the data, your chi-squared values and the Site Description cards, determine which of the cards accurately described the site for which you have gathered data.

<b>Table 1. Adirondack Mice Populations</b>					
<b>Site</b>	<b>Year</b>	<b>Population Size</b>			
		<b>Deer Mouse</b>	<b>White-footed Mouse</b>	<b>Woodland Jumping Mouse</b>	<b>Meadow Jumping Mouse</b>
Site 1	Year 1	200 of each Each represents 25% of entire population			
	Year 2	198	210	190	202
	Year 3	205	199	203	193
Site 2	Year 1	200 of each Each represents 25% of entire population			
	Year 2	70	80	107	115
	Year 3	90	100	130	140
Site 2	Year 1	200 of each Each represents 25% of entire population			
	Year 2	180	180	90	175
	Year 3	185	190	125	205

**DATA AND OBSERVATIONS**

**PART I: CHI-SQUARED TEST**

NULL HYPOTHESIS

Table 2. Chi-Squared Test for Rolling a Die							
	1	2	3	4	5	6	Total
Percent chance (%)							X
Expected (e)							
Observed (o)							
Difference (d = o - e)							
Difference squared (d <sup>2</sup> )							
d <sup>2</sup> / e							
$\chi^2 = \Sigma(d^2 / e)$							

Degrees of freedom: Number of categories - 1 = \_\_\_\_\_

**DATA AND OBSERVATIONS (CONTINUED)**

**PART II: POPULATION SAMPLING**

NULL HYPOTHESIS

<b>Table 3. Random Sampling with Beans</b>				
Animal	Total in 10 Sample Sites	Average Counted (Total / 10)	Total Population Size	
			Estimated (Average × 100)	Actual (From Teacher)
Deer mouse				
White-footed mouse				
Woodland jumping mouse				
Meadow jumping mouse				

<b>Table 4. Chi-Squared Test for Random Sampling</b>					
	Deer Mouse	White-footed Mouse	Woodland Jumping Mouse	Meadow Jumping Mouse	Totals
Percent chance (%)					X
Observed (o)					
Expected (e)					
Difference $d = (o - e)$					X
Difference squared ( $d^2$ )					
$d^2 / e$					
$\chi^2 = \Sigma(d^2 / e)$					

**DATA AND OBSERVATIONS (CONTINUED)****PART III: ADIRONDACK MICE**

<b>Table 5. Chi-Squared Test on Adirondack Mice Populations</b>		
<b>Site</b>	<b>Calculations Between...</b>	<b>Chi-Squared Value</b>
1	Year 1 to Year 2	
	Year 1 to Year 3	
2	Year 1 to Year 2	
	Year 1 to Year 3	
3	Year 1 to Year 2	
	Year 1 to Year 3	

**ANALYSIS****PART I: CHI-SQUARED TEST**

- Using Table 2 and the chi-squared result from Part I, write a conclusion statement summarizing your results. Be sure to restate your null hypothesis, chi-squared value, and statistical probability when making this statement.







**CONCLUSION QUESTIONS (CONTINUED)**

3. Mrs. Lowery bred her corn snakes so she could have baby snakes. The parents or P1 generation are “double hets,” which means that they are heterozygous for both traits that determine the color in corn snakes. In the F1 generation, four normal-colored, three grey, two red, and two snow-colored corn snakes hatched.

Table 6. Color Combinations in Corn Snakes			
Key to Genes		Key to Color	
B	Black	B_R_	Normal
b	Not black	B_rr	Anerythristic (grey)
R	Red	bbR_	Amelanistic (red)
r	Not red	bbr	Snow

- a. State the null hypothesis.
- b. Determine the chi-squared value and the validity to the null hypothesis.



**CONCLUSION QUESTIONS (CONTINUED)**

5. In a given population, the evolution of genes can be evaluated using the Hardy-Weinberg equation. The Hardy-Weinberg equations were applied to the inheritance of hitchhiker’s thumb, a recessive condition.
- A population was sampled and it was determined that 24% of the population has this recessive condition. People with the dominant phenotype have straight thumbs.
- a. Use the Hardy-Weinberg equation to calculate the “expected” frequency of each genotype.
- b. Each individual in the population was identified and labeled according to their genotype. In some cases, the parents of individuals had to be contacted to verify the label. In a population of 400 people, 160 were homozygous dominant, 144 were heterozygous, and 96 were homozygous recessive. Using the calculations from the previous question in addition to these observed numbers, calculate the chi-squared value for this data.
- c. As a result of your findings, would you conclude that evolution is taking place in this population? If so, name a mechanism of change that could be causing this chi-squared value.

**GOING FURTHER**

Research other sampling techniques that can be used when counting organisms in an ecosystem.

**REFERENCE**

Degrees of Freedom	$p = 0.9$ (9 in 10)	$p = 0.5$ (1 in 2)	$p = 0.2$ (1 in 5)	$p = 0.05$ (1 in 20)	$p = 0.01$ (1 in 100)	$p = 0.001$ (1 in 1000)
1	0.016	0.455	1.642	3.841	6.635	10.827
2	0.211	1.386	3.219	5.991	9.210	13.815
3	0.584	2.366	4.642	7.815	11.345	16.268
4	1.064	3.367	5.989	9.488	13.277	18.465
5	1.610	4.351	7.289	11.070	15.086	20.517
6	2.204	5.348	8.558	12.592	16.812	22.457
7	2.833	6.346	9.800	14.067	18.475	24.322
8	3.490	7.344	11.030	15.507	20.090	26.125
9	4.168	8.343	12.242	16.919	21.670	27.877
10	4.865	9.342	13.442	18.307	23.209	29.588

The value  $p$  is the probability that the results could be due to chance alone. The numbers in parentheses below each value of  $p$  restate  $p$  in terms of chance, e.g., a 9 in 10 likelihood that the results could be due to chance alone.

ITEM 1 - KEY TO MICE



Deer Mouse  
*Peromyscus maniculatus*



Meadow Jumping Mouse  
*Zapus hudsonius*

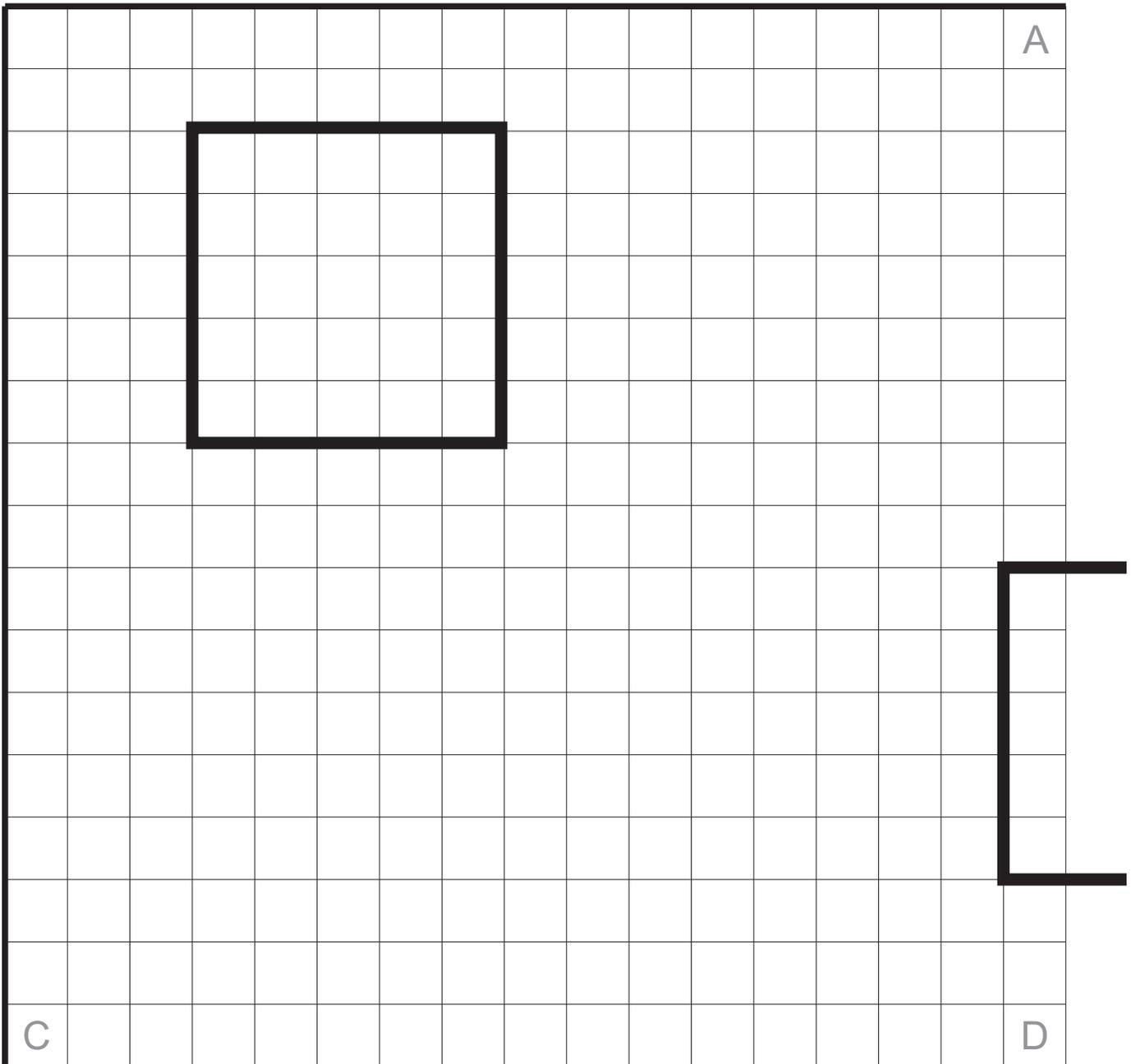


White-footed Mouse  
*Peromyscus leucopus*



Woodland Jumping Mouse  
*Napaeozapus insignis*

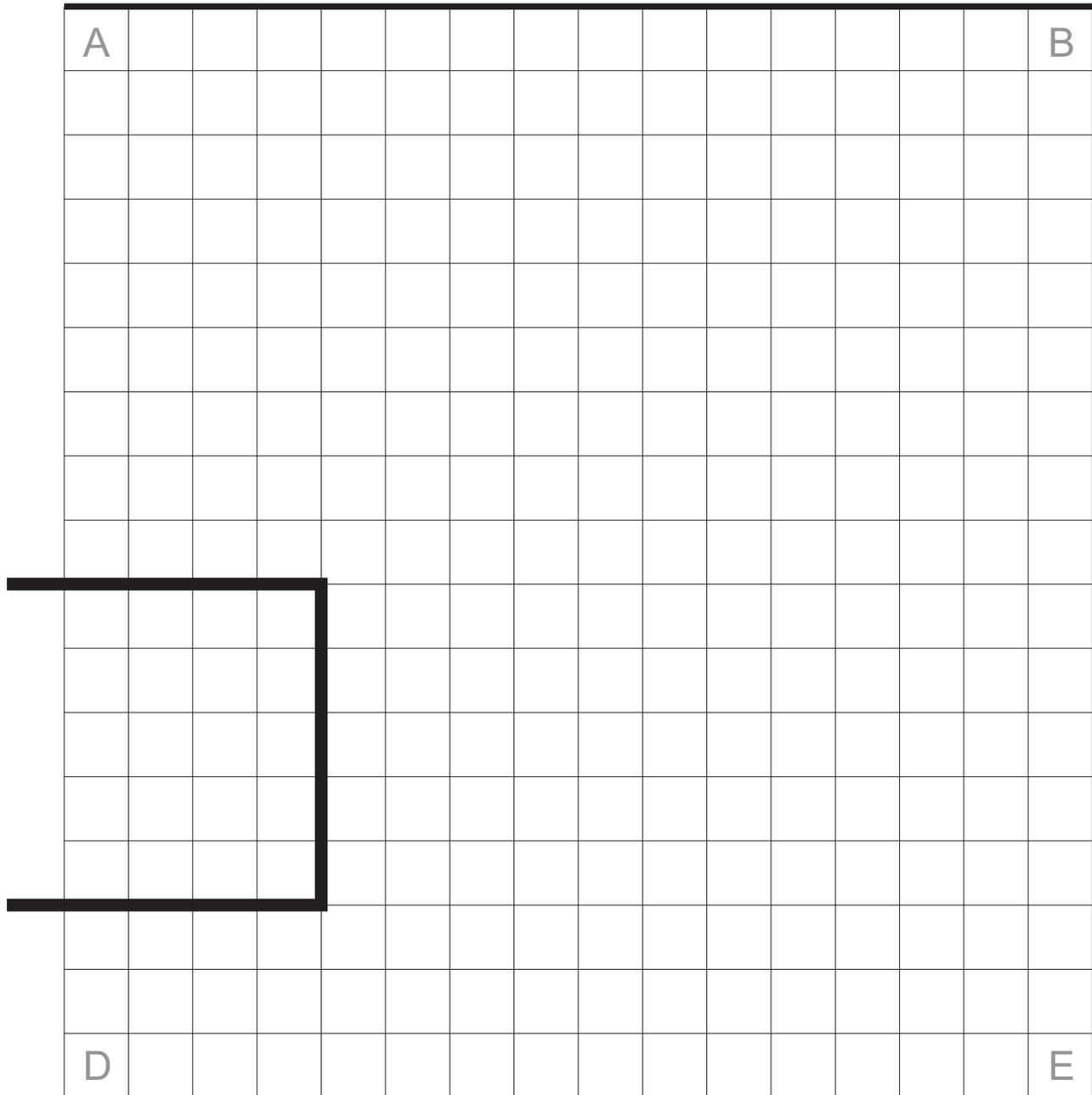
ITEM 2 - GRID PIECE (1/9)



LESSON CONSUMABLE

This image is based on a 1:1 scale. Do not shrink or enlarge this image.

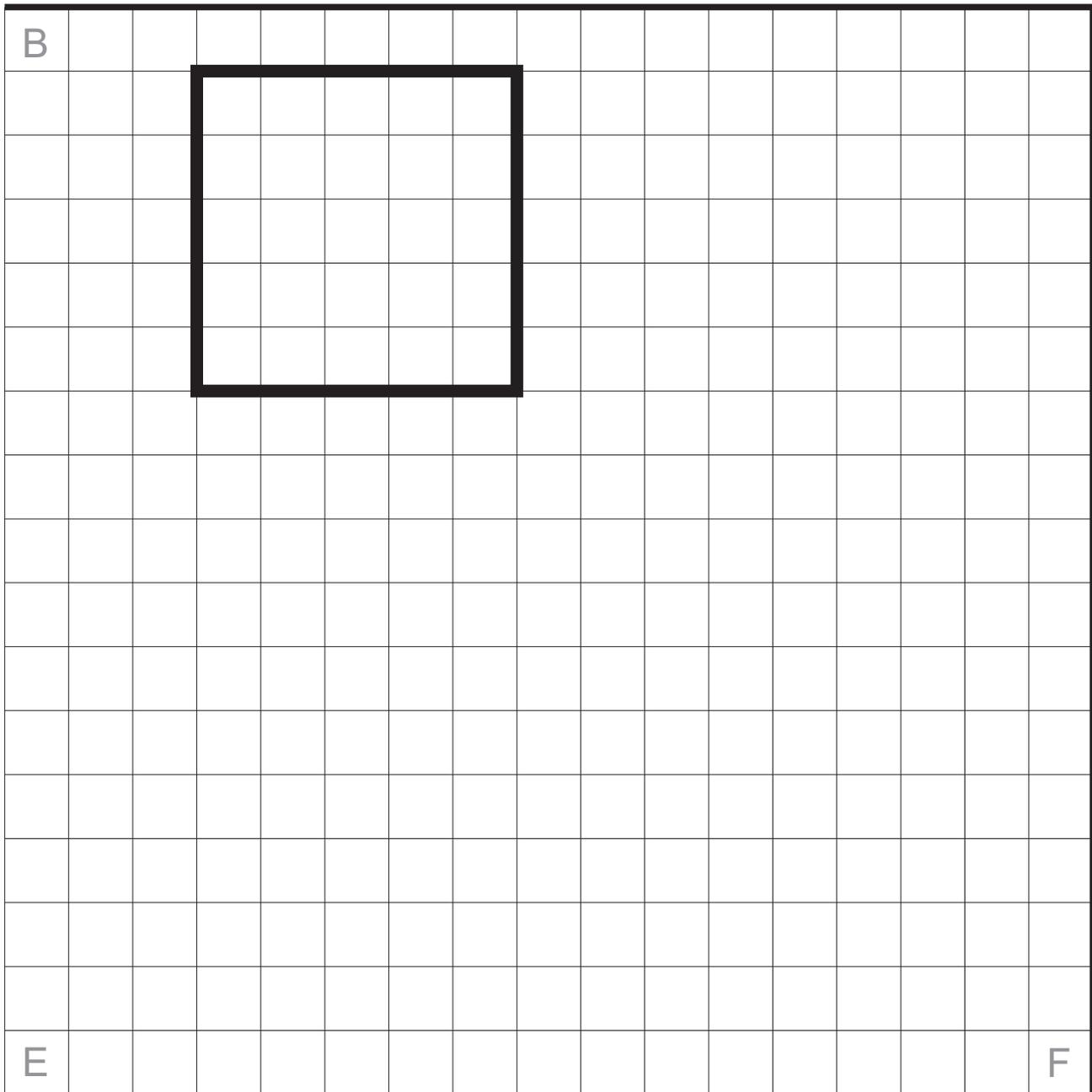
ITEM 3 - GRID PIECE (2/9)



LESSON CONSUMABLE

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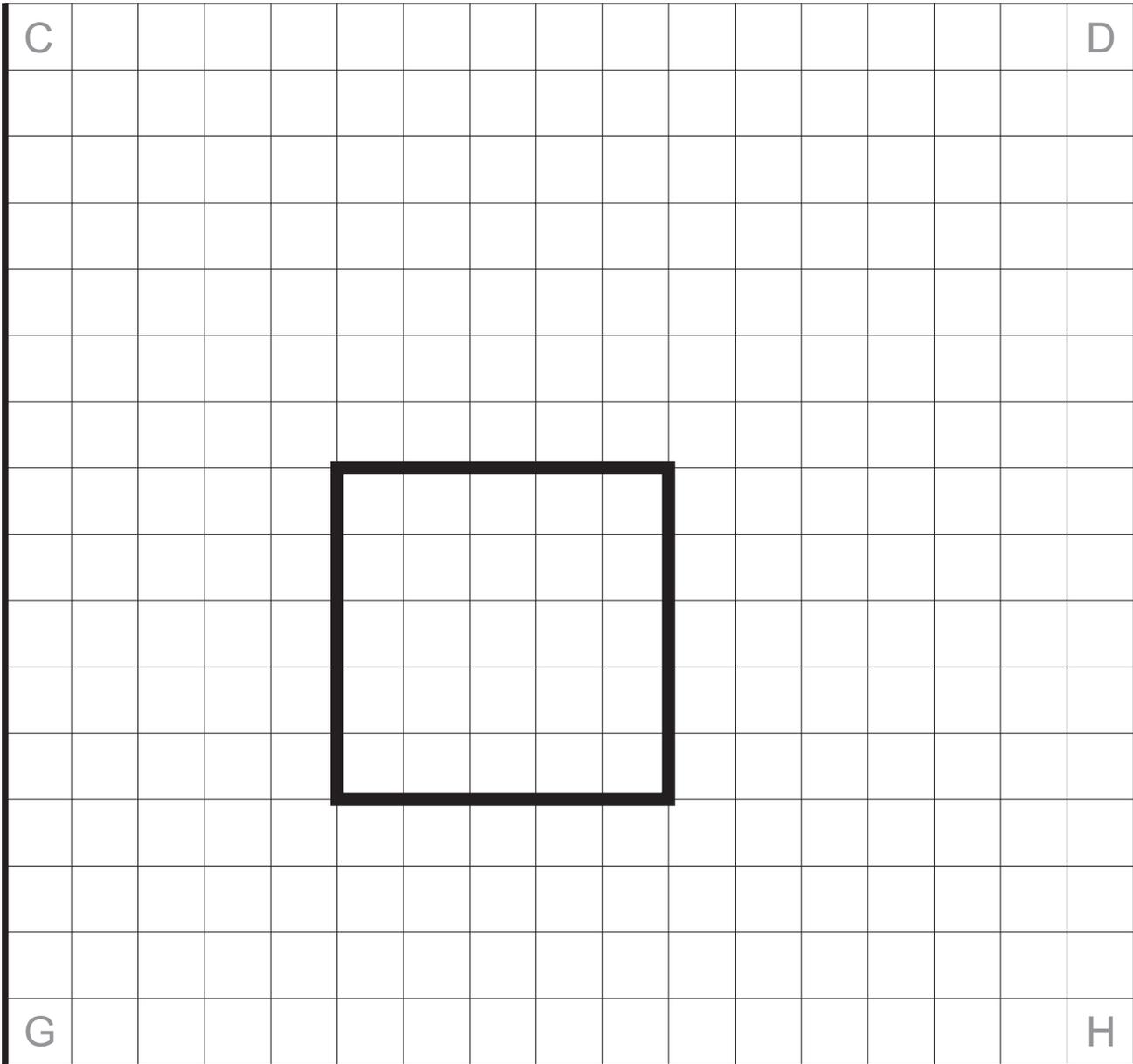
ITEM 4 - GRID PIECE (3/9)



LESSON CONSUMABLE

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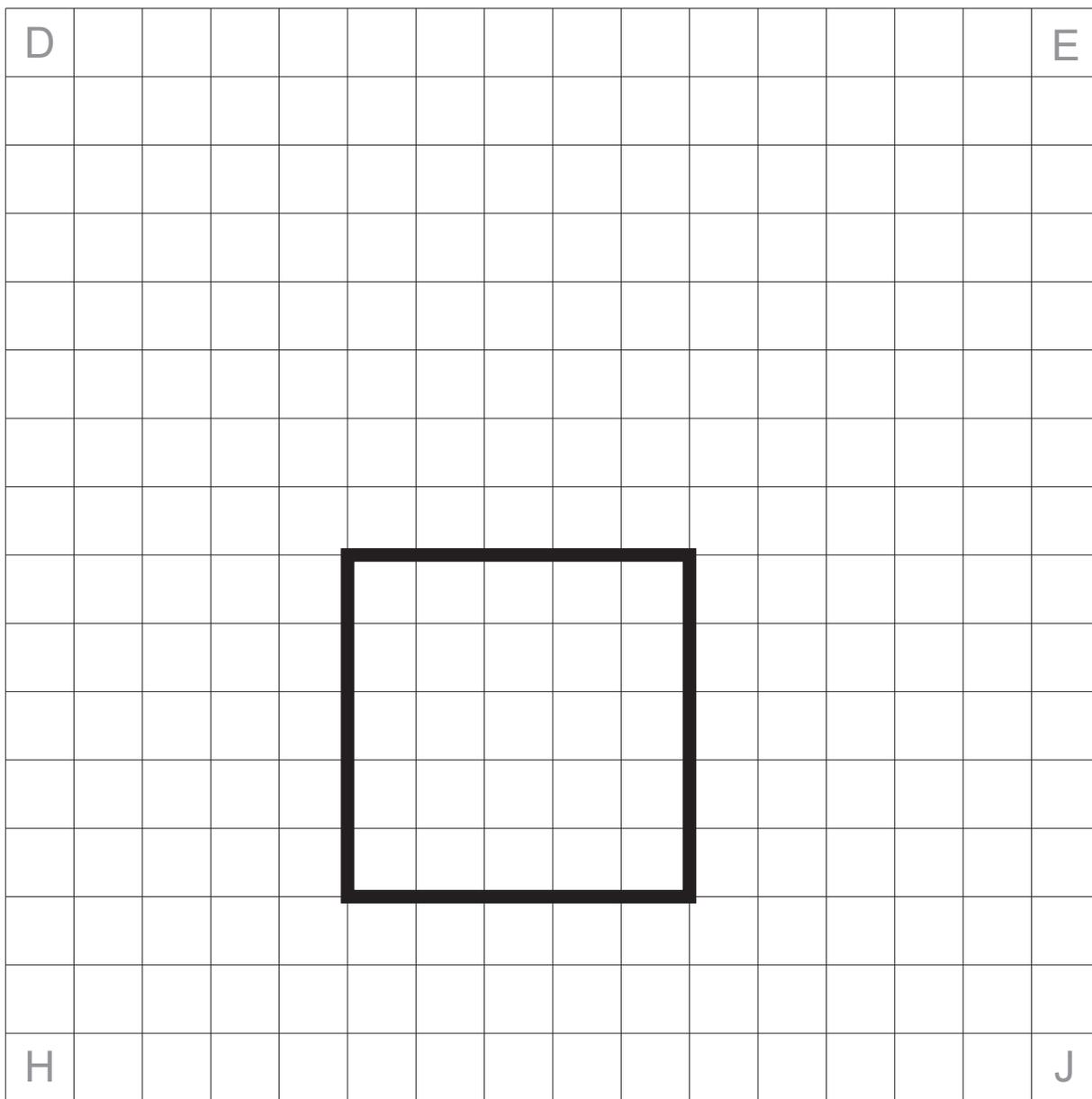
ITEM 5 - GRID PIECE (4/9)



LESSON CONSUMABLE

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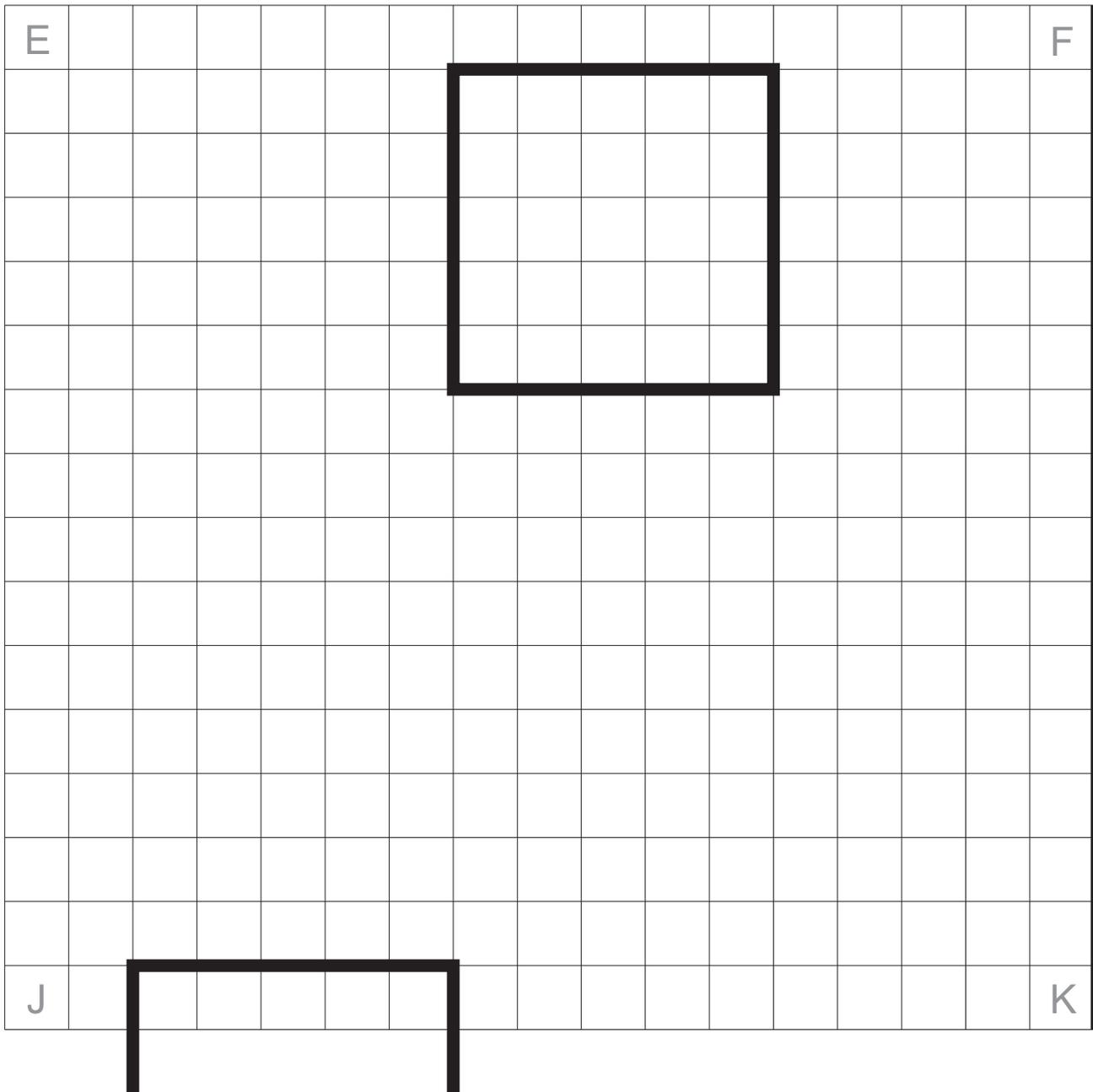
ITEM 6 - GRID PIECE (5/9)



LESSON CONSUMABLE

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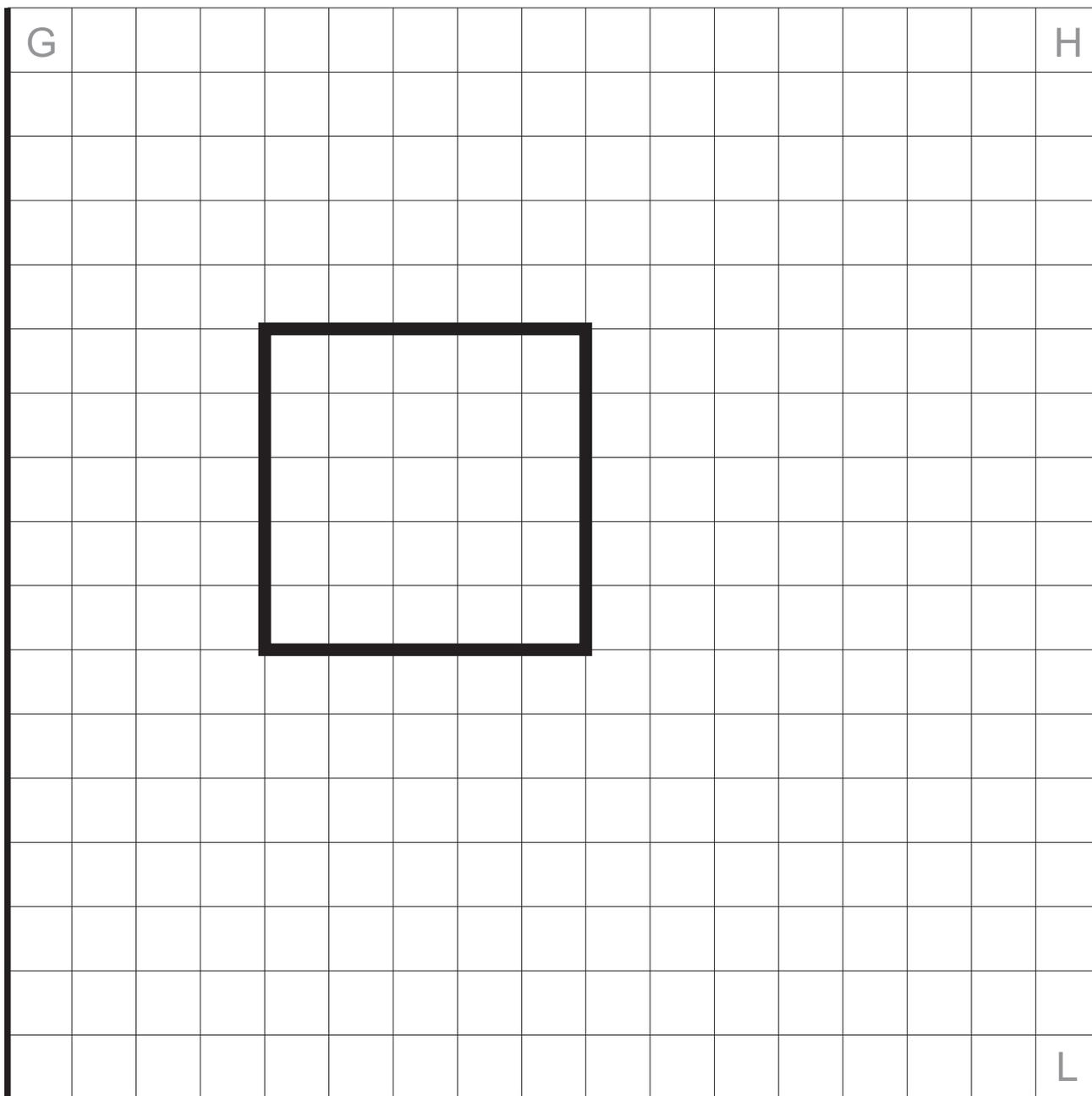
ITEM 7 - GRID PIECE (6/9)



LESSON CONSUMABLE

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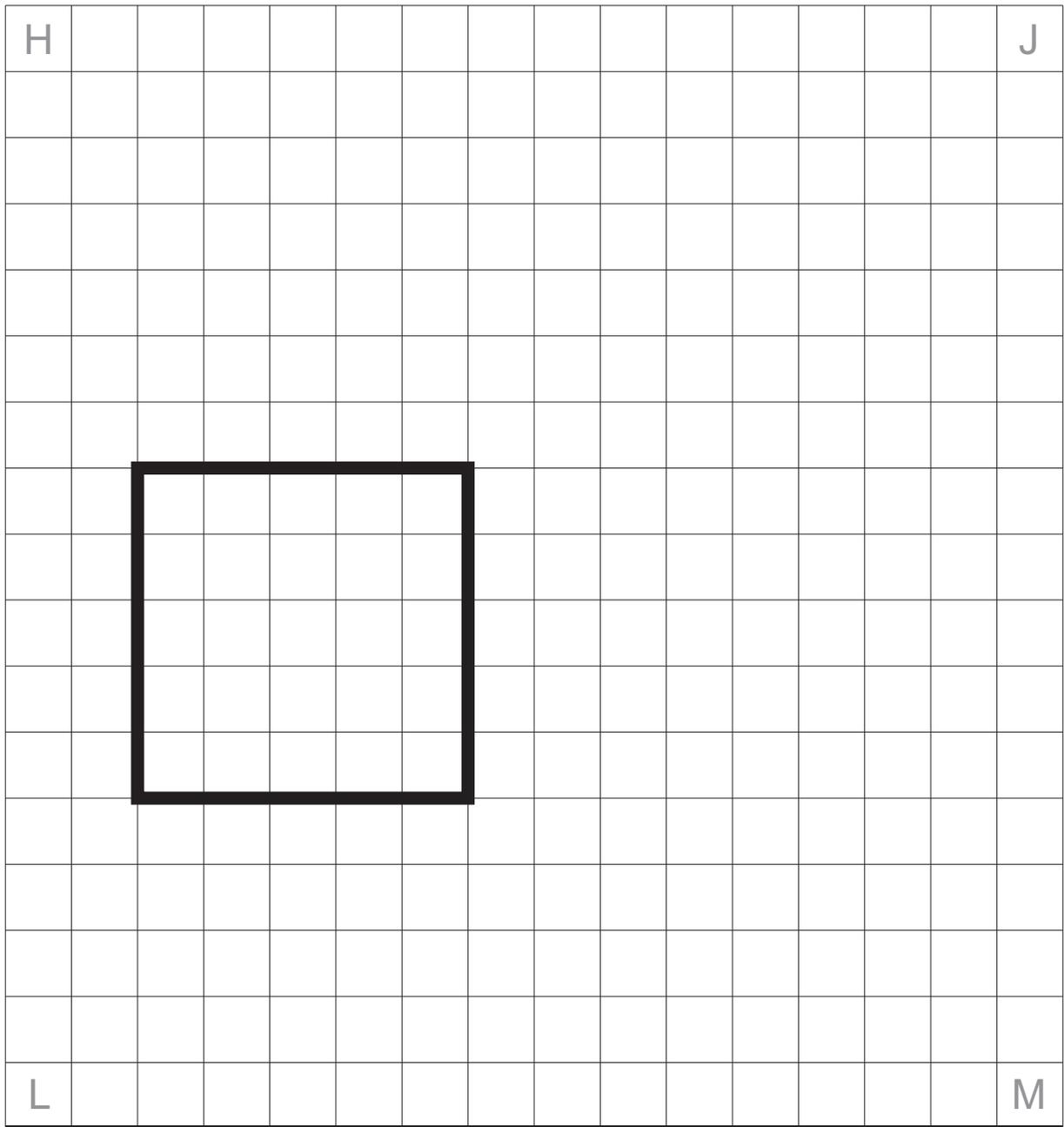
ITEM 8 - GRID PIECE (7/9)



LESSON CONSUMABLE

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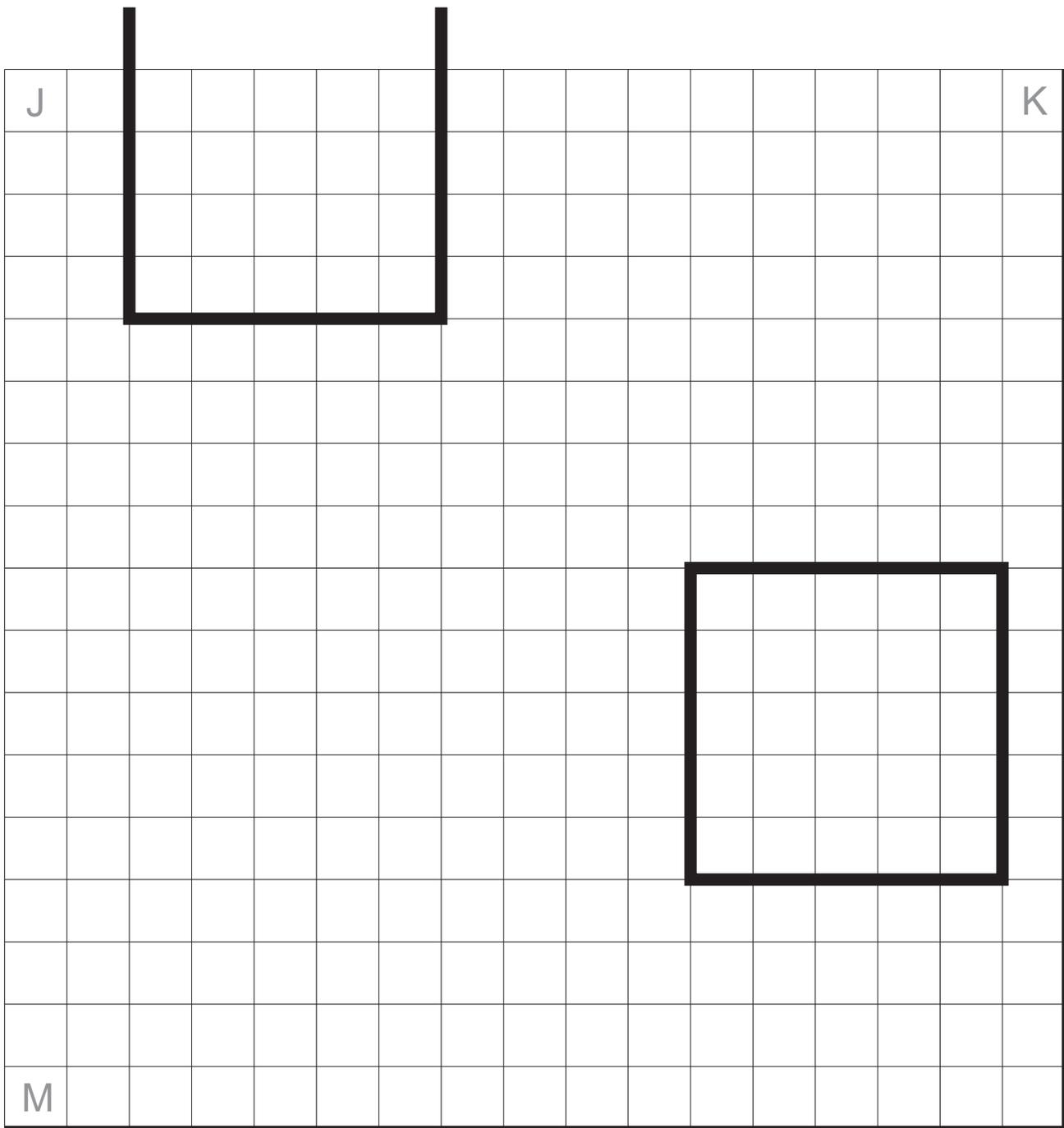
ITEM 9 - GRID PIECE (8/9)



LESSON CONSUMABLE

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ITEM 10 - GRID PIECE (9/9)



LESSON CONSUMABLE

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## ITEM 11 – SPECIES DESCRIPTION CARDS (1/2)

**Deer Mouse*****Peromyscus maniculatus*****Appearance**

Soft fur, colored back, faint darker strip, and a dark tail with a white tuft at the end

**Size**

184 mm and weight of 21 g

**Habitat**

In the Adirondacks, the deer mouse is widespread. They live in terrestrial habitats that are abundant in trees and ground cover. It occurs in all elevations.

**Diet**

Deer mice eat fungi, fruit, and even carrion. They cache food, especially seeds in holes in the ground, tree cavities, and bird nests.

**Activity**

Deer mice are nocturnal but most activity takes place at twilight. Their usual means of locomotion is walking or running. Leaping takes place when pursued.

**Reproduction**

Breeding takes place from March through October. Each female gives birth to an average of 5-6 young.

**Predators**

All predators of small mammals prey on deer mice, including hawks, owls, snakes, foxes, minks, weasels, bobcats, and coyotes.

**White-footed Mouse**  
***Peromyscus leucopus*****Appearance**

The general color is reddish or orange. A dark brown stripe occurs along the middle of the back from the head and tail. The tail is shorter, paler, and does not end in a tuft of white hairs.

**Size**

176 mm and weight of 21 g

**Habitat**

The range extends all over North America and into Canada. They are found in forests, bogs, and swamp edges at the lowest elevation of 484 m but more abundant at higher elevations.

**Diet**

White-footed mice eat nuts, seeds, fruits, and small animals in addition to insects and fungi. They cache food under a log, in bird nests, or abandoned burrows.

**Activity**

White-footed mice use underground sites more than deer mice. When alarmed, they drum their feet rapidly.

**Reproduction**

Breeding takes place from spring until fall. Each female gives birth to an average of 4 young.

**Predators**

Predators include snakes, owls, hawks, skunks, weasels, foxes, and coyotes.

## ITEM 12 - SPECIES DESCRIPTION CARDS (2/2)

**Meadow Jumping Mouse**  
*Zapus hudsonius*

**Appearance**

Longer hind legs than front legs with a tapered tail. The coarse fur is yellowish-brown with a dark midline. Large ears, dark eyes and long whiskers.

**Size**

215 mm in total length including the tail and weight of 19 g

**Habitat**

Meadow jumping mice occur in meadows, fields, forest clearings, and edges, and are also found on the edges of swamps, marshes, and bogs at elevations of 700 m and occasionally to 1220 m.

**Diet**

Meadow jumping mice eat seeds, fruit, fungi, and small invertebrates. They harvest seeds by climbing stems or bending them over.

**Activity**

When startled, can jump up to 1 m with each leap. It moves by walking, climbing, and swimming. Activity mostly takes place at night but more so during cloudy, rainy weather. Hibernation takes place from September to late October.

**Reproduction**

Breeding begins immediately after hibernation ends. Females give birth to average of 5-6 young.

**Predators**

Predators include minks, coyotes, weasels, snakes, foxes, red-tailed hawks, owls, large frogs, and carnivorous fish.

**Woodland Jumping Mouse**  
*Napaeozapus insignis*

**Appearance**

Large hind limbs, bright colors, and a long, tapered, white-tipped tail. Brighter colors and fewer teeth, and the white-tipped tail distinguishes it from the meadow jumping mouse.

**Size**

Average length is 204-256 mm and weight of 26 g

**Habitat**

Woodland jumping mice are a common resident of coniferous, deciduous, and mixed forests in the Adirondacks. It is found in the Adirondacks up to elevations of 1189 m.

**Diet**

Almost a third of their diet consists fungi from under leaves and debris. Seeds, fleshy fruits, leaves, and small invertebrates make up the remainder of the diet. This species does not cache food.

**Activity**

Woodland jumping mice move by walking or hopping, but can leap up to 2 ft in the air and 3-6 ft in distance. It also can swim but only briefly, and usually avoids water.

**Reproduction**

Females bear litters in June and late August. Each female gives birth to average of 4-5 young.

**Predators**

Predators include birds of prey (especially owls), snakes, some weasels, and bobcats.

## ITEM 13 - SITE DESCRIPTION CARDS

**Site Description - #1**

The lowest recorded temp in the Adirondacks took place this year.

The record low reached -34 throughout much of the area.

Populations of wildlife that were unable to find shelter were tremendously impacted. Insect populations were reduced to their lowest levels in years.

Flora with thick bark was able to withstand the record breaking temperature but had reduced seed production the following year.

**Site Description - #2**

Temperatures throughout the Adirondacks were on average with the exception of the mild winter. Precipitation levels increased from years past an average of 1.2 in per month.

As a result, seed bearing trees increased their production of seeds which appealed to the bear and mouse populations throughout the mountains.

As a result of the increase in moisture, an increase in allergies was noted for those with allergies to fungal spores.

**Site Description - #3**

This year brought a colder than normal winter in addition to an already drier year. Precipitation was down an average of 1 inch per month.

An increase in rodent numbers has been seen throughout towns in the Adirondacks. In addition, more tourists have reported an increase of bear sightings in camp grounds.

For those small mammals that have not a surplus of seeds or food stored, it was an extremely harsh year.

**Site Description - #4**

Temperatures and precipitation throughout the Adirondacks was right on with the past 15 years.

An increase in the Hanta virus cases was noted this year. Suspect was drawn toward the mouse populations of the Deer Mouse and White Footed Mouse.

Ecologists began a Stop The Spread Campaign to educate those that frequent the campgrounds about the spread of the virus.